CLASSICAL TORSION AND AIST TORSION THEORY

Background

The design of a crane runway girder has not been an easy task for most structural engineers. Many difficult issues must be addressed if these members are designed properly. One of the least understood subjects is the effect of torsion from the lateral forces exerted by the trolley. Before clicking on SDC Torsion, which is based on Warping Torsion Theory, we provide some background on how torsion is classically addressed along with how AIST Technical Bulletin 13 addresses the issue of torsion.

Scope

This paper will limit the discussion to girder cross sections with flanged open sections. Although torsion effect must also be considered for members with a closed section, they are treated in a different manner with a different approach.

This paper will assume that the girders are simply supported for both torsion and flexure. A simple support for torsion is such that the flanges are free to warp but not twist at the support point. Freedom to warp for a flanged member means the top flange is allowed to move relative to the bottom flange in opposite directions. This can be distinguished easily from pure flexural minor axis bending behavior for which both the top flange and the bottom flange move along the same direction in unison. Most crane girders are designed on this basis.

Excluded from this paper is another type of simple support which exists mathematically but cannot be easily achieved physically. This type of support exists where the flanges at the ends are free to twist but not warp. Even if this type of end condition could be constructed, it will be difficult to justify its effectiveness for lateral thrust load transfer to the column.

For members with simple end conditions subjected to a concentrated moving wheel load, it should be apparent that the longitudinal fiber stress is critical near the load point and the flexural shear stress is critical near the support of the member with the wheel load groups near the mid-span. Assume the load is applied at mid-span, whenever stress is mentioned in this discussion, we mean either at the mid-span of the girder or at the girder support unless noted otherwise.

To simplify further, load due to member self-weight and a source other than the crane wheels are excluded. Finally, all the deformation response due to load is small relative to the member sizes for all dimensions.

A complete treatment of torsion is by all means beyond the scope of this discussion. Our intent is to try understanding this subject from a structural designer's viewpoint.

For comparison purpose only, one cannot help but to mention flexure along with torsion. However, as simplification of the subject, this discussion does not assume that there is any interaction between these two types of stress conditions.

Torsional Stress

Pure torsional shear stress [τ_1] appears everywhere along the transverse axes of a member. Within any section, except for the local stress concentration near elemental fillets where the web(s) meets the flange(s), its magnitude is directly proportional to the shear modulus, the thickness of the profile section and the [θ'] function. In a global sense, for a simply supported girder, it is qualitatively similar to flexural shear stress, since the maximum value appears near the supports but diminishes non-linearly toward the load point.

Warping normal stress [σ_x] acts normal (perpendicular) to the section profile and so its orientation is parallel to the longitudinal axis of the member. It is critical usually at points of interest on the girder cross section that have a large "offset" from the shear center. That is why for flanged girders it is often evaluated at tips of the flanges where greater warping normal stresses are found rather than at web where offset from the shear center is hardly significant. Warping normal stress is directly proportional to the normalized warping function at the point of interest, the Young's modulus, and the [θ''] function. In a global sense, for a simply supported girder, its peaks and valleys are similar to the fiber stress pattern of a flange subjected to weak-axis bending.

Warping shear stress [τ_2], by its name, is a form of shear stress traverse along the elements of the section profile. It exists wherever there is warping normal stress and is proportional to the warping static moment at the point of interest, Young's modulus, and the [θ'''] function, but is also in inverse proportion to the element thickness. Therefore, thinner element experiences higher stress but it is always zero at tip of flanges. In a global sense, for simply supported girder, warping shear stress peaks at the load point and then diminishes non-linearly toward supports but it will not go away to zero at the supports. One interesting point to make here is that, the usage of "flexural static moment" for calculating the horizontal shear stress distribution in flexure is similar to the usage of "warping static moment" here. What's the difference? Application for flexural property deals with geometric centroiod while application for torsion deals with shear center.

Classical Torsion

Whether for an equal-flanged or unequal-flanged member cross section, this method is so much simplified that it is "applied" to both cases and it has no bearing on the location of shear center. For any member of depth [d] subjected to torque [M_z] about mid-depth, it calculates the resultant force couple acting at the extreme fiber of the flanges as [M_z / d], or for that to pass through the centroid of flanges as [$2 M_z / (2d - t_1 - t_2)$].

As a simplification only for this discussion and for comparison, we will use force [M_z / d] for this does not make much numerical difference. From three different perspectives, let us evaluate the resulting stresses from this approach against that from the torsional approach (using Table 21, case 6 and Table 22, case 1e of Reference 1):

Shear Stress at Supports

The horizontal shear stress in the flange from flexure, including the form factor, is:

$$f_v = (3/2)(M_z / d\, t_f\, b_f)$$

The pure torsional shear stress:

$$\tau_1 = \left(\frac{3}{2}\right)\left(\frac{M_z}{d\, t_f\, b_f}\right)\left(\frac{1}{2 t_f / d + t_w^3 / t_f^2 / b_f}\right)\left[1 - \frac{1}{\cosh(\beta L/2)}\right]$$

If the approximation is more conservative, then [$f_v > \tau_1$], and it can be deduced to;

$$\frac{1}{1 - (2 t_f / d + t_w^3 / t_f^2 / b_f)} > \cosh(\beta L/2) \qquad \text{where} \quad \beta L/2 > 0$$

Notice from this expression, the left-hand side is a function in terms of profile dimensions and the right-hand side is a function of profile dimensions and the member length, we just can not tell which method is more conservative if we would evaluate it strictly from numerical standpoint.

Mid-span Fiber Stress

The mid-span bending stress in the flange is:

$$f_b = (3 M_z L) / (2 d t_f b_f^2)$$

The warping normal stress at mid-span can be deduced as:

$$\sigma_x = \left(\frac{3 M_z}{d t_f b_f^2} \right) \left(\frac{1}{\beta} \right) \left(\tanh \frac{\beta L}{2} \right)$$

If we assume the approximation is more conservative, then [$f_b > \sigma_x$], and;

$$\frac{3 M_z L}{2 d t_f b_f^2} > \left(\frac{3 M_z}{d t_f b_f^2} \right) \left(\frac{1}{\beta} \right) \left(\tanh \frac{\beta L}{2} \right)$$

By rearranging some terms, we can express the ratio of result from the approximation with respect to the result from torsional analysis as:

$$\left(\frac{\beta L}{2} \right) / \left(\tanh \frac{\beta L}{2} \right) > 1 \qquad \text{where} \ \left(\frac{\beta L}{2} \right) > 0$$

This proves that the <u>classical approach to torsion is indeed more conservative</u> because this expression is always valid. Or we may say that the classic approach can lead to over-design.

"How much over-design in sense of warping?"

It ranges from merely 2% for a stiff girder with $\beta L = 0.5$, to 65% for girder with moderate stiffness with $\beta L = 3$, up to 300% for a flexible girder with $\beta L = 6$.

Mid-span Shear Stress

For flexure, the mid-span shear stress is same as for the support and the expression from §10.1.1 still applies:

$$f_v = (3 / 2) (M_z / d t_f b_f)$$

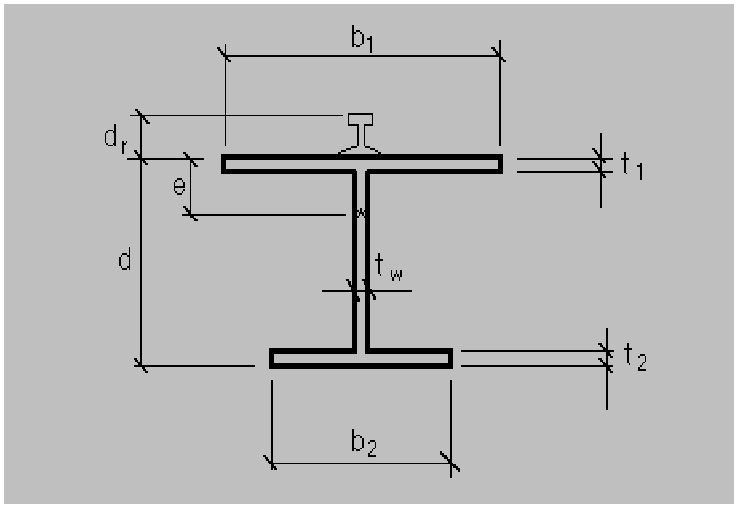
The warping shear stress at mid-span can be derived as:

$$\tau_2 = (1 / 2) (3 / 2) (M_z / d t_f b_f)$$

This is like saying [$f_v = 2 \tau_2$]. Apparently the mid-span shear stress from approximation is <u>always twice as much</u> as the maximum warping shear stress.

<u>In summary, member designed using "Classical Approach" is always more conservative than that by torsional approach.</u>

AIST Torsion



Recommended detail of this approximation can be found in Reference 2. Because the wheel load is applied at the top of the rail and also the location of shear center for runway girder is always below the top flange, calculation of torque [M_z] must be based on a moment arm, or eccentricity, of [$d_r + e$]. Forces in the flanges, after calculating from static equilibrium about shear center of the profile section:

$$F_{top} = \frac{M_z}{d} \left(\frac{d_r + d}{d_r + e} \right)$$

$$F_{bot} = \frac{M_z}{d} \left(\frac{d_r}{d_r + e} \right)$$

The only difference between force in top flange and bottom flange is the factor of [$1 + d_r / d$] and [d_r / d]. For formulas appearing in the following subsections, the subscript [$_f$] associated with thickness [t] and width [b], of a general flange, can be replaced by subscript [$_1$] or [$_2$] as reference to top flange or bottom flange, respectively.

Shear stress at support

The horizontal shear stress in flanges using AISE approach are:

$$f_{v,top} = \left(\frac{3}{2} \right) \left(\frac{M_z}{d\, t_1\, b_1} \right) \left(\frac{d_r + d}{d_r + e} \right)$$

$$f_{v,bot} = \left(\frac{3}{2} \right) \left(\frac{M_z}{d\, t_2\, b_2} \right) \left(\frac{d_r}{d_r + e} \right)$$

Correspondingly the pure torsional shear stress is:

$$\tau_1 = \left(\frac{3}{2} \right) \left(\frac{M_z}{d\, t_f\, b_f} \right) \left(\frac{1}{2\, t_f / d + t_w^3 / t_f^2 / b_f} \right) \left[1 - \frac{1}{\cosh (\beta L / 2)} \right]$$

For [β L / 2 > 0] and if AISE approach were to be more conservative, then:

$$\frac{1}{1 - \left(\frac{2\,t_1}{d} + \frac{t_w^3}{t_1^2\,b_1}\right)\left(\frac{d_r}{d_r + e}\right)} > \cosh\left(\frac{\beta L}{2}\right) \qquad \text{for the top flange, and}$$

$$\frac{1}{1 - \left(\frac{2\,t_2}{d} + \frac{t_w^3}{t_2^2\,b_2}\right)\left(\frac{d_r + d}{d_r + e}\right)} > \cosh\left(\frac{\beta L}{2}\right) \qquad \text{for the bottom flange}$$

The validity of these conditions is not so obvious and is dictated by the numerical relationship of the variables.

Mid-span fiber Stress

The bending stresses in flange per AISE approximation, including the form factor, are:

$$f_{b,top} = \left(\frac{6}{t_1\,b_1^2}\right)\left(\frac{L}{4}\right)\left(\frac{M_z}{d}\right)\left(\frac{d_r + d}{d_r + e}\right)$$

$$f_{b,bot} = \left(\frac{6}{t_2\,b_2^2}\right)\left(\frac{L}{4}\right)\left(\frac{M_z}{d}\right)\left(\frac{d_r}{d_r + e}\right)$$

Assuming that AISE approach is more conservative, then as for the "Classical Approach", we can conclude that:

$$\left(\frac{\beta L}{2}\right) / \left(\tanh\frac{\beta L}{2}\right) > \frac{d_r + e}{b_f\,(d_r + d)} \qquad \text{where} \quad \beta L / 2 > 0$$

For practical design, the value given from left-hand side is always greater than [1], and the value from right-hand side is always less than [1], the above relationship is always valid and therefore, AISE approach is always conservative.

Mid-span Shear Stress

For the shear force at mid-span for flexure is same as the shear force at support, expressions derived for shear stress at supports from §10.2.1 will apply. If AISE approach is more conservative, then for that we must have:

$$\frac{d_r + d}{d_r + e} > \frac{1}{2} \qquad \text{for top flange}$$

$$\frac{d_r}{d_r + e} > \frac{1}{2} \qquad \text{or simply;} \quad d_r > e \qquad \text{for bottom flange}$$

In practical design, apparently the relationship for top flange is always true. Condition at bottom flange is more critical, if in words for [$d_r > e$], it is valid only if "the rail depth is greater than the distance between shear center to extreme fiber of top flange".

<u>In summary, except for the toss-ups in shear stress at supports and at mid-span, the fiber stress at mid-span calculated by using "AIST Approach" is more conservative than that by torsional approach.</u>

Nomenclature

b_f	Width of flange in general
b_1	Width of top flange
b_2	Width of bottom flange
Cw	Warping constant of cross section
d	Depth of the member
d_r	Rail depth
e	Distance from shear center to extreme fiber of top flange
e_f	Rail float or eccentricity of rail with respect to shear center
E	Young's modulus of member
f_b	Flexural bending stress in an element
$f_{b,bot}$	Flexural bending stress in bottom flange
$f_{b,top}$	Flexural bending stress in top flange
f_v	Flexural shear stress in general
$f_{v,bot}$	Flexural shear stress in bottom flange
$f_{v,top}$	Flexural shear stress in top flange
F_{bot}	Force resultant applied in-plane of bottom flange
F_{top}	Force resultant applied in-plane of top flange
G	Shear modulus of member
h	Distance between the centroid of two flanges [$d - (t_1 + t_2) / 2$]
I	Moment of inertia of cross section
J	St. Venant torsional constant of the cross section
L	Member length
M_x	Bending moment about major axis of the cross section
M_y	Bending moment about minor axis of the cross section
M_z	Sum of torque moment M_{z1} and M_{z1}
M_{z1}	Torque due to vertical wheel load about the longitudinal axis of the member
M_{z2}	Torque due to lateral thrust load about the longitudinal axis of the member
P_v	Vertical wheel load applied at top of rail
P_h	Lateral thrust load applied at top of rail
r	Distance from shear center to a point of interest
t_f	Thickness of flange in general
t_w	Thickness of web
t_1	Thickness of top flange
t_2	Thickness of bottom flange
β	A torsional wave length constant defined as [$(J G) / (C_w E)$] $^{1/2}$
θ	Rotation about the longitudinal axis
θ'	Member rotation about the longitudinal axis

θ'' Member rotation about the longitudinal axis
θ''' Member rotation about the longitudinal axis
σ_x Warping normal stress
τ_1 Pure torsional shear stress
τ_2 Warping shear stress

References

1. Roark & Young: "Formulas for Stress and Strain", 5th edition, McGraw-Hill, 1975.
2. AIST Technical Report No.13