Open Sectioned Crane Runway Girders with Arbitrary Profile Geometry

Chapter 7 – Special Subjects, Rare or Inconspicuous

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As in so many cases of conducting business as usual, when solving Familiar Structural Engineering Problems by "widely received" approaches – including those not necessarily the best suited or that entirely error-free – but in the end, most of us are **OK** to proffer credence to an illusory wisdom, that is to trust whatever and however we had done, it would always turn out satisfactory results.

> On occasions, when the engineered outcome isn't as rewarding as expected then, we must be curious to find out what happened; perhaps (1) the problem being solved is not an ordinary one after all – although it may look so recognizable – or else (2) we might have misapplied our die-hard personal-professional understanding of the algorithm and approach used in solving the given problem in the first place
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> To list a few to that effect, it could be as simple or as serious as:
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> - Not realizing the "widely received" approach is not suitable or inadequate for the task,
> - Miscalculation in section properties, load magnitudes or load effects,
> - Error or omission in definition of basic loading cases or load effects or
> - From making unjustifiable assumptions, etc.
>
> Many of us might have similar experience, not all our engineered results were one-hundred percent satisfactory every time, the depth of our disappointment from which usually hinges on how on track (or off track) we had comprehended and assimilated to what were passed on to us from the beginning
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> In normal practice, all elements and articles as-mandated or as-specified in the problem/project definition should have been taken in as part of our engineering deliberation. And sometimes the initiative thereof to our cognizance are not always so clear-cut or as being one-hundred percent conventional. How things turn out with that in the end may vary with how thorough and how accurate we had deciphered the given information

Incidentally, we should have accepted it by now and by heart, in the realm of proper engineering of **Crane Runway Girders (CRG)** – based on their behavior – there is no such thing as Simple-bending.

> **So before we press the enter key or click that magic button for a quick solution, always check to see what is missing first.** The biggest fallacy in it is, many of us don't know what to check or aware of what is missing. There are subtle **CRG-unique** situations we have to pay extra attention to, not only the discernible issues but also certain subtle elements that were not so well-defined or well-understood
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> Or even when we "fantasized" that all the "issues" were resolved during and after the **CRG** engineering process, sure we can claim we did it all, but if so then on the other hand, how come there is cracked weld and sheared bolt? For that we must give honest answers when being asked;
>
> > Had we mistreated torsion?
> > Had we forgotten about fatigue?

Regardless to what kind of response, yet besides professing the verity perhaps there is incorrectness in torsion treatment and inadequate provision of strength against metal fatigue, what else is missing? Had we failed to subjugate all the other "must-get-it-done issues" under our belt? When in doubt, shall we take a closer look if we have missed any supplementary constrains with concealed characteristics that are unique to **CRG**?

Here is a disputable but rather harsh reality,

> We got by with what we did for so long without realizing (*or refuse to realize*) that, certain "widely received" methodologies and "business as usual" approaches simply cannot handle **CRGs** as to complementing the definition of "proper **CRG** engineering"

Thereby we should come to a common ground that drastic improvement is needed over our long established technical-empirical and engineering-social ways and means for the better; otherwise "proper **CRG** engineering" is nothing but a fancy catchphrase

Then beyond "business as usual" on a more sophisticated level with specific reference to justifying the adequacy and/or qualification of **Crane Runway Girders (CRG)** as a whole, for the most part in this day and age, one should "see" it right away the emerging of unusual design requirements that were only so blatant and unique to **CRGs**, like taking care of things such as torsion and metal fatigue.

And yet some/most of us might or might not "see" – as if motivated by not "seeing things" on purpose − into the significance of certain must-be-seen issues at the same eye level. Ironically in punitive term "not seeing things" is equivalent to not paying due respect to "proper **CRG** engineering".

Anyhow just so we understand, harnessing a universal slogan-like phrase such as "business as usual" into everyday **CRG** practice could contrive diverse inferences.

Promoting such widely-accepted – but not so sound – advice to the dealing with every state of **CRG** affairs would not be sustainable; i.e. if we insist on "business as usual" too often too rashly then, by and large someone (else) will "uncover" any inexcusable consequence just a matter of when− sometimes the most uncomplimentary kind

In the Mills, the worst inconvenience out of those *ill-fated under-engineered girders* should be those *untimely unpromising varieties*, there a tiny trace of of girder-introduced flaws can cause major inconvenience to the Operation, which sometimes can strike it cold and hard at the wrong moment when no one had prepared for – occurring not necessarily near the end of a girder's service life, yet – from which if short of triggering a total collapse, but quite enough a valuable lesson can be learned from any "near miss incident," *only if someone is "willing" to learn*

On **CRG** design, (1) abiding by the as-mentioned unsullied "widely received" philosophy too grossly or (2) carrying out the same old *simple-bending-based process* lacking broader attention to critical "details" and (3) not taking exception to the "simple-bending rules" could bring about substandard aftermath – sounded like a broken record but good to hear it once more anyways

Hereinabove the word "substandard" for situations if coupled with tough lucks could be substituted with the word "catastrophic" to suit, for examples:

- To certain critical components, **drastically underestimated** the critical stress state and/or their (ultimate) strength against material yielding, instability or metal fatigue – even for the less serious variety if there is – so should we question for the record, have we "seriously" qualified the structure against metal fatigue?

- Or on situation with deteriorated serviceability or performance in general – or on the much longer-termed functional or physical degradation – there we go and no one questions, how many of us had "correctly" estimated the maximum deflection at the rail top?

During **CRG** engineering-design development, should the analytical/design progression imparting "simple-bending principle" in the application dispensed with *technically flawed* or *procedurally deficient* approach, *undeveloped or underdeveloped ways and means,* or *any encoded schemes containing errors or logical bugs*, etc. − even for schemes so inadvertently adopted, inherited or acquired – or for whatever poor reasons and excuses, budding defects of various measures would for certain crop up in the structures, and it's no laughing matter.

Broken record again, one of the hard facts to always keep in mind, there is no such thing as simple-bending in the **CRG** world

Many engineers don't "see" the hurt how distressed **CRG** feels as result from wrongful application of "simple-bending principle" plus a few other technical issues if imposed; once the ill-engineered analytical-design-detailing elements were ingrained in the structure, whatever was sown up by design is already sealed in queue, and there is no way out

One of the disadvantageous positions to be in from Mills' perspective is not knowing what's good or bad ahead of time,

> Having no knowledge of the fact that they are gambling or playing the waiting game has become the norm, because most of the (major) detriments ensuing any flawed engineering would not take an instant effect but tend to be well hidden in plain sight when everything is new – until it's too late

And that is how **CRG** works

Even though degradation in performance from ill-engineering, if only predictable, might not be so widespread or entirely off target, but to those unfavorable elements already seeded in crucial components or connections thereof – *stitched welds* or *missing rail clips*, for example − if serious enough and *not tended to properly* then the worst of all are bound to flare up eventually; when that happens, it would be the very costly kind;

> Ironically be that however costly, but, almost all the times the Mills are footing the bill while the Engineers of record were either long gone or rarely be held accountable

Thus before proceeding too hastily – on finishing/signing off/issuing the design deliverables much too soon for fabrication/construction in this context – for better or worse for being "qualified" Engineers doing honest engineering work, double checking what comes out of our "business as usual or unusual" is a must and with a *refreshed well thought-out engineering judgment* always apply – for example, look for "unjustifiable" no-torsion assumption although that works exclusively for structures exhibit "simple bending" behavior;

> Then how many Engineers are "qualified" to see it correctly is the question

At times no matter how well equipped and how competent we "think" we are as to effecting good solution to a given **CRG-themed** problem – with all the engineering good-well we could contribute – we may still find the "need" of calling for technical support for various reasons, especially during forensic analysis when facing unexpected or escalated impedance, etc., so it's normal that every individual may need help once in a while – self-helped or being helped.

<u>On shorter term – with not much of choice but had to do it even for the no-fun part of it</u>:

Whenever we, being far too inferior in handling certain critical issues, sensed that (1) our **CRG** business as commissioned is not at all that usual or (2) the inherent situation isn't as conventional as it should have been or (3) the available choices of tools for a better resolution aren't too many then we might opt to:

- Get around-go around developing quick-fixed (*simplified*) solution and deal with the issues on our own resolve, that could be right or wrong, or

- Take up promptest guidance from readily-saved-up knowledge pool (through explicable collection of previously worked-out design examples or reference materials, etc.) whether applicable or not, or

- Call for impetuous advice in and around the nearest proximity (Office) or otherwise end up going nowhere, etc.

<u>On much longer term – only for better</u>:

As to solving specific **CRG-themed** problem or designing specific type of structure, not every Engineer can do it all or know how to do it all, especially covering all fine points on day one of assignment – it takes more than practices

<u>During early stage of our career</u> for lack of "solid" experience in **CRG**, we might rely on voluntarily accumulated statistics and knowledge or spontaneously learned engineering ways and means, stance or advice, etc. − be that deemed appropriate/inappropriate for the occasion, or not so certain of what acquired were accurate or inaccurate, and that whether truly tried-and-true or not – but over time the information or reference materials could have been gathered up:

- From certain rank-and-file so-and-so or being passed on earnestly from those *supposedly* highly skilled experts or mentors, or

- From prevalent or indiscriminate source at large, luckily if there is a right source

<u>Then in any case:</u>

The way (1) how we utilize the collection of **CRG-specific** knowledge into actual application and (2) how we authenticate the application's aftereffect might have profound influence on our lasting professional mindset onward as time passes

Henceforward on any and all occasions with reference to **Crane Runway Girder Design** in terms of torsion and fatigue, double checking our results and practicing with a *well thought-out engineering judgment* always apply.

7.1 Looking Back At Business as Usual

Starting with structures <u>other than</u> **Crane Runway Girders**:

From a General Engineering Analytical/Design viewpoint:

As far as recognizing, categorizing, labeling and computing the varieties of internal stresses transpired at a specific **X/Y/Z** component as in response to external load goes:

"Business as usual" tend to lead in a self-proposition prompting that those same old same-old we were accustomed to would work out just fine (if not always)

In succession, should we have premeditated our engineering approach around normal cognitive measure all the times, then our knowledge extent on the diversities and categories of internal stress associated with structural qualification purposes would be "limited" to structures exhibit behaviors conforming to **simple bending**

The so-termed "limited" would fall within the same-old as what typically slated in the opening Chapters of most 101-oriented Textbooks or rudimentary Design Guides, whereby once the "limit" takes hold, seldom is there a desire to outgrow beyond

Carrying on with the same old custom for the same old reason so logically (or illogically) into conducting same old "businesses as usual" seemed nothing wrong. Only that there is this downside, we tend to stay with the same old custom and become too strong-headed to accept new things unless being advised ahead

In the end, if not conferred on *our own voluntary causes or for consigned **R&D** purposes, or not from under unavoidable pressing situations, etc.*, fewer and fewer of us would step out of our own way to supplement further interests and resources into other makes of stresses or design situations not-so-well-publicized or those <u>normally</u> not as widely <u>recognized</u> or well <u>understood</u> subjects

Continuing on "business as usual" with respect to **Crane Runway Girders (CRG)**:

To a familiar extent *historically* owing to:

- Their unique load application nature (at top of the rail) and
- The harsh in-service surroundings (**X/Y/Z 3D** moving loads) they had to survive in, and
- As much as our *ultimate* design goal for which is to provide ample strength to stave off threats from metal fatigue among other threats, etc.

Already those were plentiful pretext to "suppose" that there might (or must) be situations not as widely <u>recognized</u> or well <u>understood</u> and yet might accrue (serious) damage onto our structures, or else *why are there so many <u>structural repair-related</u> maintenance upkeep issues – that is happening in many Mills right now*

A more fitting line of attack in **CRG** business is to always assume the business is not as usual as ever experienced. To that effect on the whole, even though some of the very seasoned yet unfazed "experts" among us might be boasting they had already seen enough of that, done it all and presumably had known it all by heart of what to do by far, but, how many of them dare to support their claims and demonstrate with calculation to prove it?

Weighing up whether to take **CRG** business as usual or not although is a personal choice, but don't make the wrong choice.

There is no need to wait for specific timing or near the end of the game to make a correct ruling; and regardless, it's already proven that <u>not every **CRG** in active service</u> can be established to function in a flawless state at all times − just because there is so much of wear-and-tear going on – and more importantly, *a matching wisdom also applies to judging our own engineering "application knowledge" and design techniques as well*

Once on our toes for more for the better, it is no longer abnormal to nurture a freshly "normalized" mental realism in **Crane runway Girder Design**, for the least:

- To reflect further than what seemed aplenty to take in already, be prepared to explore beyond **simple bending** and deal with unusual design state of affairs outside of normal set of internal stress and implication from various blends of structural responses, and

- To realize no other design situation is worse than dealing with **shear center-triggered** issues while messing with **unsymmetrical section properties**, etc.

Consider from a bigger-picture perspective:

- On defense: With respect to structures already in service for a long period of time − first and foremost is to get familiar with the past (or the latest) deficiency-findings logged in the (historical) inspection report(s) especially those entities that went through rounds of up-and-down series of repair maintenance-function

    Despite the service performance being good, fair or poor, etc. as rated by the Mill's Maintenance panel, and it's not a matter of who does or doesn't trust whom, but for being short of key facts, just take it by face value first

    In a way we should not indiscriminately embrace an all-inclusive position on Engineering Quality "thinking" all is well and perfect and can hold out forever without a qualified engineering-driven evaluation

    Being dutiful Engineers on our duty, we should look into the reason(s) why certain critical components – base metal or stitch welds, for instance – failed, and shouldn't *speculate* the answer without backing it up by calculation

  Take a typical **CRG** withstanding harsh operating environment as usual, from which one should be able to single out an as-detailed component at certain **X/Y/Z** locality and make an assessment based on the critical stress level

  If the component was not qualified (through proper calculation) with adequate strength against specific mode(s) of failure then sooner or later an "incident of distress" would come to pass whether if concealed for time being or already logged in the Inspection Reports – *In certain Mills, Maintenance Crews are constantly on their heels for good reasons*

- On offense (and defense, too): To us Practitioners even if seasoned through numerous Engineering Design (training) sessions:

  Despite for ages of doing the same old same-old, whatever **CRG** engineering approach had we acquired and accustomed to, just as we think that nothing big is going to change or happen, then comes along the need of renewing our engineering-analytical techniques due to usual and/or unusual challenge embedded in many new (newer) site-specific engineering ventures – *Text Books or Design Specifications are getting thicker for good reasons*

The catches upon us continuing onward:

> There were more of the unfinished puzzles to be exposed and to be put together, more of the modern design principles and regulations to be developed and to keep up with, more of the improved methodologies to be fostered and to follow, and more of the better-understood theories yet to learn from, etc.
>
> All that could have (1) flourished into up-to-the-minute directives drawn from ever-evolving **R&D** conclusions/consequences, or else (2) learned the hard way from catching up, patching up or making up with age-old design negligence or from renewed retroactive learning while fixing up to mitigating longstanding engineered mistakes, and so forth – easier said than done

All for sake of continuous improvement, what observed bolting down onto our inner reading chamber's entryway are those unstoppable modern waves of design mandate – for testimony, simply compare the thickness of the Design Codes and Standards issued back then and right now − one ought to acknowledge and accept the fact that these newer rules being put into effect do actually enhance, modify or replace the old ones, or whichever already proven deficient, too vague, too out-of-date or no longer apply, etc.

7.2 Maybe Business Is Not Usual

Deliberating from Mill Maintenance's perspective,

> Consider the <u>negative aspect</u>: take a somewhat battered structural entity and critique its condition based only on (recorded) evidence of:
>
> - The (mounting) <u>number/count</u> of findings of structural deficiencies/distresses or excessive (permanent) deformations as called out in the Inspection Reports and
>
> - The (growing) <u>number</u> of (recurrent) structural repair work orders or events as logged in maintenance record
>
> Here we are looking at a beat-up structural entity; provided we have access to the "official" records, from which it should give a "fair" indication even from a laymen's point of view as to whether practicing "our old way" of "business as usual" that does work or does not work for these girders − especially the ones of unsymmetrical section – so we are the judges; but no matter, facts are facts that need no Engineering-based cover-up, confession, consensus or denial, etc/

Speculating from an engineering viewpoint,

> Based on "fair" evaluation of structural performance as recorded in the most recent inspection reports and/or survey results – what went on (or what went wrong) with past engineering in this regard should be very clear and wouldn't need to be reintroduced again and again
>
> If only critiquing based on review of <u>engineering calculation</u> in general, one would agree that other than advancement in automation-related hardware/software tools, it hasn't changed much in how **CRG** were treated "structurally" from looking back on engineering work done half a century ago or at least through last few decades as of this writing
>
> *The biggest issue is:*
>
> > ***CRG** should have been treated with mechanical functioning and long-lasting performance in mind beyond all things pure structural*
> >
> > That is to say then and now at the time of this writing, not much of in-needed elements were added onto the "practical ways and means" in the design of **CRG** against **mechanically induced** loads on the whole – in particular on moderating structural deformation and preserving the in-service longevity of *unsymmetrical sectioned crane runway girders*
>
> The notion of bringing up or considering (1) *mechanical performance-driven loads* and (2) *long term load influence* to structural detailing design seemed like a novel subject, actually it's not
>
> > Be on that issue, forget about however scanty attention the subject had earned (or not earned) from the mainstream in the past, just let it be. For now instead of shunning off, giving up, evading or looking the other way on **Unsymmetrical Sectioned Crane Runway Girders'** behalf, come what may, it's more important and sensible to be technically open-minded and be ready to draw a new picture, the rosy picture
>
> At this point we are on our own beyond simple desire if that's for some long overdue knowledge into dealing with **Unsymmetrical Sectioned Crane Runway Girders**. *Regardless to what took*

place beforehand, the key issue is how do we blunt out the risk from future design flaws or engineered failures; in other words, how to be a part of the solution and not part of the problem

The point in the making:

> *Structural Engineers look at Runway Girders' material, stress and strength as pure structural entities while the Mill Maintenance Crews look at them by functionality as an undivided component of the mechanical system*
>
> A little switch over from of our traditional way of thinking leads to the renewed understanding of a long-standing design mandate: Once a structure is to survive the mechanical way of life, naturally we need to deal with **fatigue**

As far as **Crane Runway Girders** are concerned, the all-inclusive aspect of engineering involvement in design of which had been proven far past "business as usual" and the truth to that could be demonstrated whenever dealing with their **Repair and Upgrade** aspects – thankfully so for that's where most of the newer challenges come from – business is not usual

7.3 Looking Beyond Business as Usual

Furthering the objective brought about at beginning of **Chapter 6**:

Tending to normal and primary stresses is already a significant part of our normal and primary design responsibility;

But there might (or must) be other "acquired occurrences or load effects or responses" (as if) *not so normal* or *not so primary* but are jointly "remitting" their pinching-punching-tearing-shearing here, there and everywhere in **CRG's** everyday life yet mostly ahead of us that need to be examined

Granted, both secondary stresses and local stresses subsist in all classes of structures, of which certain varieties might be established as conventional or as unconventional from a certain (non-) mainstream point of view

Then again those inexplicable kinds, whether as being part of our **CRG** design concerns herein – *make a judgement only after a few pages ahead* − or in other **non-CRG** applications, could be well concealed (or else deemed unimportant or negligible) and thus mostly were unaccounted for or ignored − as if not in existence or otherwise still unsettled among many of us unsuspecting of their presence

In any case, **secondary stress** and **local stress** may cross each other at some locale in some ways but they are of two different effects;

Both entities have distinctive significances although may share similar features in terms of turning out shearing, bending, twisting, stretching or contraction effect onto an element, a fiber, a pocket or a specific plane across, etc.

Each entity could be established sparsely, simultaneously and/or independently from different loading events as immediate consequences or as spinoff effects, and that could be instigated either in different cross sections, zones or components of the structure, or both could be coexisting at the same locality, etc.

This **Chapter** places focus on selective varieties of those effects, some of which takes place quite obvious in the Mills yet not so obvious on the *engineering desktop*, that in turn may subtly contribute to upsetting the design outcome of our **CRG** to certain degree − depending on how well the structure can absorb or put up with the striking on the spot and carry on into longer service term besides enduring the ever-dominant assault from primary stresses.

By usual judgement:

> Just because being (already) ruminated “secondary” or “localized” short of further discretion, these effects were often insinuated (*unintentionally*?) into something “minor,” “trivial,” “miscellaneous,” or “not well recognized,” etc. or somehow opined generally as trivial entities – those easily forgotten, slipped through the cracks or overlooked by certain branches of design mainstream – especially by the naysayers
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> Quite commonplace among many Practitioners, it seemed nothing wrong with thoughts drawn on wisdom in parallel with the foregone judgments. Yet at times those rationales could be the reason why these matters were not included “aptly” in regular **CRG** engineering practice routines, for which nowadays geared “so much” or “so exclusively” towards the “primary” global loads, global structural behaviors or global design requirements
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> Once again, effects deemed collectively as “secondary” or “local,” whether misguidedly or rightfully so classified, might not always translate into the notion of “unimportant” or “simple” to certain classes of structure − although some phenomena might be *not so well understood, unconventional or plain old inconspicuous, etc. but are in need of much needed attention* − win or lose, these effects do join hand with other conventional effects and deliver serious harms to our structural components

Finally, technical treatments especially to these obscuring topics could incur fairly boring derivations, yet so tedious enough that it may lead the way to some formulations not necessarily that complex but rather user-unfriendly if not simplified for hands-on application.

7.4 The Risk of Doing Nothing or Not Enough

In certain applications, disregarding entirely the so-called secondary effects or any specific type of localized stress’ presence might depend on whether:

- The seriousness of effect, intensity of stress induced, the locality and frequency of occurrence were much more or less “threatening” than that from other forms of primary effects or stresses; but the problem is how can anyone prove such effect is “not threatening” or just the other way around if without calculation, and/or

- It was proven (really?) their effects on the overall structural design outcome and the in-service performance were indeed negligible (again, don’t buy that without proof)

> Easy to say we made the choice of discounting those “effects/conditions” based on best judgment, but that is the hardest part − one should not draw quick conclusion from false equivalence between what are normal in **non-CRG** applications and that not so normal in **CRG** applications in this regard; there could be veiled risks from not taking into account those effects on **CRG** behalf – ignoring or not is best be settled on careful evaluation
>
> Nevertheless,
>
>> *The level of unfriendliness* and *seriousness* imparted to any specific structural component/element/connection due to selected secondary and/or local stress is not that straightforward to catch on without “detailed” calculation – not just any calculation but the kind drilled on competence into whatever to be calculated

In other words, the structure cannot be fully qualified prior to a thorough review of stress combination results by adding together the secondary and primary stresses − as we shall see how the secondary effect may play its role in the upcoming example

A similar frame of mind should apply to the appraisal of final design outcome of structure taken as a whole including all critical connection details as well – would any of that be affected by the inclusion of local/secondary stress is what the design Reviewer should answer before signing it off

Knowing that these effects/conditions do exist, but on emphasizing the importance of whether taking them in seriously or not for argument's sake, it could be technically misleading and/or legally contentious at times, depending on:

- Which side one chooses to stand behind, and under what rules to stand by as to representing the defense or the offense, siding *politically, legally or technically*?

- The timing factor − when were the conditions or issues evaluated or re-evaluated, confirmed at present, a year ago or after, or 50 or many more years ago?

- *Most importantly, the devil is in the detail; where is the (missing) calculation? − If tensile-stress fluctuation or shear-stress reversal were <u>suspected</u> to present at an inconspicuous hot spot then what is the local fatigue strength allowed thereof under the condition?*

Much as expected, depending on which component is under assessment, the controlling factors is in how has it been detailed and where exactly is the component located

To that specific component,

In which as the shred amount of local/secondary stress in it were compared with the sheer quantity of primary counterparts, and both were measured by the respective numerical significance proving the relative intrinsic value is indeed out of proportion between the two, then in a way lending good "potential" for local/secondary stress to be downgraded or ignored

For sure that "potential" may work out in conventional applications − especially <u>**when metal fatigue is not one of the design concerns**</u>

But in order for **Crane Runway Girder** to live a long or much longer life, "*design against metal fatigue"* and "doing the chore correctly" should be the first few things we the Engineers are to be concerned with − so that the *occurrence of metal fatigue* would become the last thing Facility Owner had to dwell on. Sometimes a trivial 1 ^{ksi} can make or break, just so we know doing the chore correctly/accurately or not have bearing on how inclusive/exclusive we are as to dealing with secondary stresses.

On sustainability of any individual **CRG**, what were logged in in the in-service maintenance records and performance score in the report cards could be immensely affected by the *engineering bottom line figures* if only we could trace back to the "original" calculation – whether done with or without including these "secondary" effects.

In other words, our engineered bottom line figure prescribes the structural endurance limit against <u>fatigue tensile stress fluctuation and/or shear reversal</u>. So now the question on treatment to these "secondary stresses" may linger:

At which point could anyone articulate between "do" and "don't" with an unfaltering confidence? When making the right choice, it's either

Yes - they could be ignored entirely or simplified selectively, or

No - these effects should not be ignored at all

<u>When metal fatigue becomes one of the general design concerns</u>, the "right choice" may depend on (1) whose interest is at stake and (2) what type of structural entity is at risk – girder, tie-back, support column, or foundation − and/or which structural connection or component is in the focus.

Be upfront with what we are dealing with is important because some of these effects of concern may have a lot to do with a *specific type of stress category, a special support-boundary restraining condition, a certain loads-resultant pointing along specified orientations or their coupling with geometric imperfections or the effect of other varieties, etc.*

<u>The bottom line once more</u>: Other than material yielding, the adequacy of individual component falls on how unforgiving is the fatigue stress fluctuation/reversal being allowed for the specific component as detailed or as engineered, and of course the overall quality as-fabricated and as-built as well.

What advocated herein is to always speak, defend and judge with hard numbers;

For instance, given from calculation an absolute stress value, say <u>as little as</u> $5^{\text{ ksi}}$ or $3^{\text{ ksi}}$ that seldom will or may never will alter the passing grade (structural qualification) under most **non-fatigue evaluation** results

But, the potential (or the actual) killer is the reversible (from positive to negative) sign these numbers may carry, if that is true then the "fluctuation upshot" upon combining that with fluctuation of primary stresses (from other load events or combination source) may present a detrimental measure causing the component to fail the final exam when assessed per fatigue strength qualification requirement

To further the point of reasoning why we should press on everything numerically:

Consider a certain brand of inconspicuous (secondary) stress as calculated, whether at magnitude of $5^{\text{ ksi}}$ or $3^{\text{ ksi}}$, the distribution pattern of which could be considered "not so primary" or "not so widespread" although it would be difficult at times by looking at the digit 5 and numeral 3 to demarcate what are primary and what not so primary in terms of level of influence to the final design outcome

Any stress intensity – both longitudinal and shear– can play numerical hide and (no) seek from one component to next or to nearby component, especially when coming to the inherent uncertainty in the true structural performance (see **Example 7.1**) and its subordinate relationship with the actual **fatigue allowable stress ranges**

There is no easy way of knowing <u>ahead of time</u> whether the structure (and components thereof) will last to the end. An improper qualification of some of the prescribed local connection detail(s) may (or may not) end up damaging not only the local component(s) but also the overall structural integrity in the long run, therefore one needs to really watch out for any irrational rationales or bogus **assumptions** made during various analysis/design stages

It is good to know in certain **CRG** encountering,

Some of the load response effects <u>were presumably well understood</u> for ages already but <u>were in fact misunderstood</u> all along whereas the very same effects were sensibly included or mandated in selected practices at one point in time, but, sporadically ignored or "unaccounted for" (on purpose?) at a different instance − as if it doesn't (never) exist with no explanation given

Anyhow, several subject stress categories as demonstrated hereinafter could be inspiring when appreciated with an open mind if only as for information purpose or under inquisitive intent, except for the notion on their existence and the "duration" if they ever so existed in such manner, which may be debatable but that is for the Readers, Official **R&D** or the Code committees to confirm

The objective herein is to skim the surface hopefully thoughtful enough to bring up interest without incurring full-blown **R&D** efforts, and for the simple cause that just in case there is curiosity in deciding whether these stresses could be "ignored" (or not.)

7.5 Top Flange Local Bending Stress – Derivation

Speaking of the dispersion of stress, in a much broader sense, the term "local," as opposing to "global," does not necessarily mean "secondary" or "unimportant."

But from a global design perspective, since the relative extent of stress was so "localized" then, its effect should be "fast dissipating" and its scope of coverage should be "not wide-spreading."

Starting with the "clean" base metal of a typical **CRG** member for instance,

No matter if the materials acquired were flawless, the as-built structure from which as qualified, and its connection details as designed were fabricated free from blemishes such as cracks, dents or some other form of acquired imperfections, yet local stress could still take shape from other forms of disturbance out of various sources, other than from fabrication, erection, processing, operation or maintenance but from additional reasons such as:

- Inherent discontinuity in the local geometry and/or the material, or the restraining effect near the supports or certain connections nearby, or
- From application of "concentrated" load, "concentrated" bending moment, torque or displacement owing to compatibility, or
- The interaction at interface with attachment, external machinery or accessories, etc.

In any event,

The "top flange local bending stress" of specific interest herein is not of the same category as that due to tensile forces recognized per current **AISC J10.1** provision (as of this writing) − which deals mostly with the loads applied at the flange(s) of a member owing to framing crossing point with other member(s) involving hanger-like pulling actions

By one's imagination of the result from tensile pulling under the **AISC J10.1** design intent for an **I**-shaped member, the "pulling" could have pried, bended or flipped the outstanding top (and/or bottom) flange elements over and away from the web element that tends to augment the interfacing angle between the two elements (from less obtuse into more obtuse.) Such tensile pull would induce simple plate bending about the local **z**-axis of the beam flange(s)

Yet the attention herein in **CRG**-specific interest is the bending not about the local **z**-axis but about local **x**-axis passing through the mid-thickness of the top flange, which is propping directly underneath the rail in vicinity where the crane wheel concentrated load is applied (See **Chapter one** for orientation of **XYZ** system)

The concern is neither about (1) mining into the reason why the relevant "stress" exists or (2) digging up the logic of why its existence was flip-flopped from dissimilar mindsets in recent years only had one paid attention to the issue, nor in debating if the "stress" is unimportant or insignificant to the design of **CRG** structure, but to delve into the topic and find out "where," "how" and "why" it "may" exist **not only in the girder top flange but also in the crane rail above**.

Although the general focus of the Article Series is on runway girders with arbitrary profile geometry, but for sake of "simplifying the matter in hand," say, we were given the set of initial parameters and a number of initial design conditions for a typical singly symmetric-sectioned **CRG**:

Both ends were simply supported

> E = Young's modulus of steel
> I_x = Girder moment of inertia about X-axis
> L = Member length
> P_y = Vertical wheel concentrated load over the crane rail/girder top flange
> a = Distance from the left support to the load point
>
> Furthermore, concentrated load "$\mathbf{P_y}$" (1) is pointing into gravitational axis passing through the **shear center** and (2) it does not cause undue torsion; while "$\mathbf{P_y}$" is co-linear with the girder web and thus the girder section as a whole would only deflect downward along the loading direction

The total downward displacement observed at top flange would be the combination from both "global" and "local" effects:

> The member's integral deflection under load point due to **global** effect is simply:
>
> $P_y\, a^2 * (L - a)^2 / (3\, E\, I_x\, L)$
>
> It corresponds to global bending stress based on the familiar formula;
>
> $\mathbf{(M_{xa}\, C / I_x)}$ where:
>
> M_{xa} = Strong axis bending moment at the load point
>
> C = Perpendicular distance measured from girder **X**-axis through **elastic centroid** to a point of interest on the section profile

However, in order for the "integral girder section" to **fully** resist the concentrated load "$\mathbf{P_y}$" prior to the "eventual global effect" to fully take place,

> Both top flange and bottom flange must react in unison – naturally, such a dual-flange engagement can be realized only through the web that "struts" vertically in between
>
> As matter of fact during "$\mathbf{P_y}$" load transferring process, the probable mode(s) of failure in the web, if any, from such load model were consistent with the design intent per current **AISC J10.2** through **J10.4** (as of this writing) on failures from *web local yielding, web local crippling and web sidesway buckling*, etc.

Consider an infinitesimal "unit **z**-segment of web" being pressed down (compressed) by axial load "$\mathbf{P_y}$",

> During a transient state, the narrow web segment along **y**-axis would have behaved "locally" as a strut or as an equivalent **Euler** column prior to the full engagement with the girder spanning lengthwise (this is only an assumption on the local **z**-segment but most importantly the girder itself must be stocky enough along the **z**-axis so that buckling is precluded)

With these parameters:

> h_w = Clear web height between flanges

t_w = Web thickness (column depth)
b_0 = 1” (unit **Euler** column width)

At this point, what we don’t want to deal with is the global lateral torsional buckling thus the prerequisite is lateral torsional buckling is preclude only if the girder is stocky enough

Due to the presence of <u>much wider width</u> of flanges that is **x**-oriented along the web’s **perpendicular** trace, the web segment of nominal length of $\mathbf{h_w}$ is “*constrained*” at both ends against lateral **x**-sway **locally**

Let’s say in <u>simplification</u>, but just for now, that so-called “*constrained*” is equivalent to being “*fixed*” so long as the flange-web interface angle remains at 90 degrees

Certainly one could argue that the word “*fixed*” may be too optimistic in general but the condition should hold provided that the girder is strong enough – as we said − in preventing it from being (1) lateral-torsionally buckled and (2) sidesway buckled during the loading stage

The effective length of the “web element” if treated as an **Euler** column would be ($\mathbf{k_e\, L_c}$) where:

$\mathbf{k_e}$ = effective length factor and
$\mathbf{L_c}$ = the length of the column

The “**theoretical**” value of $\mathbf{k_e}$ for a truly fixed-fixed member = 0.5, which when applied to the column length $\mathbf{L_c} = \mathbf{h_w}$ in this case would become exactly half of the actual column length (or the effective length = $\mathbf{h_w / 2}$)

The axial contraction of this column now becomes the **local** displacement of interest.
From **Hooke’s** law:

δ = Column axial contraction of web
= (P_y * length) / (cross sectional area * E)
= (P_y * ($\mathbf{k_e}$ h_w)) / (b_0 * t_w * E) and taking $b_0 = 1$
= (P * h_w) / (2 * t_w * E) where $k_e = 0.5$

For calculation of column axial stiffness:

P_y = Unit load
= 1

K = Column axial stiffness
= 1/ δ
= (2 * t_w * E) / h_w or
= (t_w * E) / (k_e h_w) where $k_e = 0.5$ for now

Noticed that the column width $\mathbf{b_0}$ = 1” is measured along **CRG** longitudinal **z**-direction

As crane wheel travels from one end to the other end of girder, the 1” web wherever that (**z**) is actively considered under load $\mathbf{P_y}$ = 1 would experience an instantaneous contraction by a constant amount “δ” provided that the girder web were of <u>uniform depth and thickness</u>

Whenever a concentrated point load $\mathbf{P_y}$ is applied through top flange (having thickness $\mathbf{t_f}$) of an **I**-beam,

The web segment beneath the flange would develop not *concentrated* but flared out under the load point as if *distributed* reaction causing axial (vertical) deformation over a localized region that has a **zone** width usually <u>much wider</u> than $\mathbf{b_0}$ = 1”

If the vertical deformation in web were taken at a spot at certain distance away from the load point, then the deformation as measured would dissipate rapidly from within the **zone pocket** in several conspicuous wavy ripples (alternating upward and downward switching the numerical sign as it goes) but would be on decreasing amplitudes in succession that would subside at a distance relatively further away from the **pocket**

Being elastic, the distribution pattern of reaction measured away from the load point would be directly proportional to the axial web deformation **but** that would likely be in a nonlinear fashion near the zone where load "$\mathbf{P_y}$" is applied

This said condition fits well with that of "beams on elastic foundation," in which the web acts as elastic foundation, which brought forth with its own equivalent "sub-grade modulus" whereas the combination body of "rail and top flange" would function as the "equivalent beam" of interest.

Refer to **Roark's** Chapter on "**Beams on Elastic Foundation**" and define terms as follows:

I_x = Moment of inertia of the equivalent beam about X-axis

K = Foundation sub-grade modulus
= Stiffness of equivalent column spring
$= (t_w * E) / (k_e h_w)$
$= (2 * t_w * E) / h_w$

b_0 = Beam width
= unit 1"

$\underline{\beta}$ = Characteristic foundation-beam stiffness ratio parameter
$= (b_0 K / 4 E I_x)^{0.25}$
$= ((2 * t_w * E) / h_w) / 4 E I_x)^{0.25}$
$= [t_w / (2 I_x h_w)]^{0.25}$
$= [t_w / (4 k_e I_x h_w)]^{0.25}$

To avoid any confusion:

Pay special attention that the characteristic parameter $\underline{\beta}$ in this regard has been <u>underscored</u> herein and hereinafter to tell it apart from another "β parameter" appeared in this Article Series that were meaningful mainly in the world of torsion

Notice that the bending moment in the equivalent beam is a function of $\mathbf{P_y}$, $\underline{\beta}$ and several other parameters

Before going into the detailed moment formulation, we could let:

α = Moment coefficient

M = Total beam moment
$= \alpha P_y / \underline{\beta}$
$= \alpha P_y (4 k_e I_x h_w / t_w)^{0.25}$
$= \alpha P_y (2 I_x h_w / t_w)^{0.25}$

Since the Crane Rails were placed over the **CRG** flange and were stabilized along all **3 Dimensions**:

- Through direct vertical wheel contact pressing against **y**-axis
- Through contact friction along **x**/**z**, and
- Through intermittent rail fasteners along **x**/**z**

Besides, rail fasteners were normally prescribed with snug fitting at regular **z**-intervals along rail length intending to clamp down and guide the rail from (excessive) uplift and (excessive) lateral movements

In engineering-mechanical sense,

> Unless intentionally having both the rail and girder integrated (why do that?) through positive means other than using rail clips into one unit as part of a functional design – *for being physically independent and non-binding*, rail fasteners do not transfer any flexural horizontal shear (the **VQ / It** kind) that otherwise would have developed at the interface between the rail and the top flange − *thus the rail does not actually participate in the formation of composite action in any ways with the girder (through its top flange)*

However, when girder is under vertical load that is pointing downward:

> With rail being sandwiched in between the crane wheel and the top flange, all that must deflect downward together and must conform as close as to the **general** (global) **CRG** system curvature (deflection shape)
>
> Without "positive bonding" from composite action otherwise, the rail and top flange together would behave as "layered beams" but still making contact locally when subjected to global flexural bending under the wheel

In other words, *the effect from "**total beam moment M_x**" would be resisted by an "algebraic sum" of moment of inertia rather than their "composite sum"*

If maintaining the parameter "α"as provisional regardless to whichever/whatever it might turn out to be later on then it can be factored out independently as a detached placeholder for generic cause; and thus the local bending stress, $\mathbf{f_{bfc}}$, in the top flange could be expressed in terms of girder/rail properties; with additional attributes defined as follows:

b_{ef} = Full flange width or post-buckled effective top flange width whichever is applicable

*The consideration of effective width here is consistent with the local buckling implication per **AISC Table B4.1***

t_f = Top flange thickness

I_{ef} = Effective moment of inertia of top flange
$= b_{ef} * t_f \wedge 3 / 12$

I_r = Moment of inertia of rail

I_{xx} = Sum of moment of inertia
$= I_{ef} + I_r$

d_r = Rail depth

M_f = Moment in top flange
$= M_x (I_{ef} / I_{xx})$
$= \alpha P_y (I_{ef} / I_{xx}) (4 k_e I_{xx} h_w / t_w)^{0.25}$
$= \alpha P_y (I_{ef} / I_{xx}) (2 I_{xx} h_w / t_w)^{0.25}$

M_r = Moment in rail
$= M_x (I_r / I_{xx})$

$= \alpha\ P_y\ (I_r / I_{xx})\ (4\ k_e\ I_{xx}\ h_w / t_w)^{0.25}$
$= \alpha\ P_y\ (I_r / I_{xx})\ (2\ I_{xx}\ h_w / t_w)^{0.25}$

f_{bfc} = Flange local bending stress
$= M_f * (t_f / 2) / I_{ef}$
$= \alpha\ P_y\ t_f\ (4\ k_e\ I_{xx}\ h_w / t_w)^{0.25} / (2\ I_{xx})$
$= \alpha\ P_y\ t_f\ (2\ I_{xx}\ h_w / t_w)^{0.25} / (2\ I_{xx})$

Being treated as unknown, the moment coefficient "α" is a function of:

- The point of interest where moment is evaluated
- The boundary conditions and
- The extent of beam length, which may be infinite, semi-infinite or of finite length

To be consistent with our **CRG** notation adopted in the Article series, herein we would replace the notation appearing in **Roark's** closed form solutions:

- From **Roark's** "x" to "z" for coordinate and
- From **Roark's** "W" to "P_y" for load term

*To avoid confusion, one should take notice of how **z** coordinate is defined, which is measured as offset from the* <u>*point of load application*</u> *– not measured from the left support – that could be anywhere along girder span*

Depending on the length of equivalent beam as summarized in **Roark's** there are three basic categories of general solution for beams on elastic foundation problem.

In essence the applicability and complexity in the solution formulations had very much to do with the beam span and the stiffness of the supporting medium (equivalent soil) under consideration, or more specifically with the rate of attenuation of responses (deformation, shear or moment)

The rate of attenuation of responses depends on the value of β**L** where:

$\boldsymbol{\beta} = [(t_w / (4\ k_e\ I_x\ h_w)]^{0.25}$
$= [(t_w / (2\ I_x\ h_w)]^{0.25}$

L = Girder length

The expression of generic solution formula for beams of "finite length" bears a much more complex look because none of the structural responses (moment, shear and deflection, etc.) would attenuate in rate as rapid as that for beams of semi-infinite or infinite length.

Whenever the beam has an β**L** value approaches or exceeds **2π** (or roughly 6.28 or rounded down to 6.0,) its structural responses near the load point where **z** = 0 are pretty much the same whether considering it of finite or infinite length.

7.6 Top Flange Local Bending Stress – Numerical Example

Example 7.1

A typical **CRG** application given these parameters:

$k_e = 0.5$

Length **L** = 25'

= 300”

h_w / t_w = 160 for a compact (or non-compact web?)

ASCE 100# rail I_r = 44

Effective top flange of 2” X 36” for that I_{ef} = 24,

Calculated $\beta L = 300 * [(1 / 2 / 68 / 160)^{0.25}]$
= 42.2, which is >> 2π.

Then consider these two conditions:

(a) Beams of infinite length

Boundary conditions: both ends extend continuously to infinity

Coordinate z = 0 under the load; (**Roark’s** infinite beams Case 10)

$M = (P_y / 4 \beta) (e^{-\beta z}) (\text{Cos } \beta z - \text{Sin } \beta z)$

There are several ways to review the structural responses at any given **z**-coordinate (measured as offset distance from the load point)

Normally one would predefine the β constant, plug that in with desired **z** value into the formula and figure out the **M** then call it good

Since the bending moment formula is a function highly dependent on **z** and β such that their collaboration would always lead to a unique α value, but the resulting value could be confusing or misleading to the eyes of beholder if one has no idea of what and how that **z** is related to the member length **L**

Luckily help is on our side in that for (1) when βL is much greater than 2π and (2) for the general treatment to condition with both exponential and sinusoidal functions in the mix, we could get by and be better off if doing it the **normalized** way without bothering **L** but by playing with various combinations of **z**, β and π as we would do next

When z = 0,
$\alpha = 0.25$ (or ¼)

$z = \pi / 4\beta$,
$e^{-\beta z} = e^{-\pi/4}$
= 0.20788,
$\alpha = 0$

$z = \pi / 2\beta$,
$\alpha = -0.052$

This condition applies aptly to wheel located far away from the girder ends (supports.) Notice that the maximum moment coefficient $\alpha = 0.25$ occurs at the load point where **z** =0. Substituting this value to the formula:

$$f_{bfc} = P_y\, t_f\, (2\, I_{xx}\, h_w / t_w)^{0.25} / (8\, I_{xx})$$

The value of "α" attenuates to 0 as **z** approaches **π / 4β**. However for **z = π / 2β** and beyond, it becomes negative and then dissipates at further distance

Interestingly when wheel travels within a short distance from **z = 0** to **z = π / 2β**, the extreme fiber in the top flange at an interior point of the girder can experience a longitudinal stress reversal up to a range of:

$$\alpha_{range} = 0.25 - (-0.052)$$
$$= 0.302$$

(b) Beams of semi-infinite length

Boundary conditions: left end free and right end continuous to infinity

<u>Coordinate z = 0 at left end;</u>

1. Apply load P_y at z = 0

$$M = -(P / \beta)(e^{-\beta z})(\text{Sin } \beta z)$$

When z = 0,
$\alpha = 0$

$z = \pi / 4\beta$,
$\alpha = -0.3324$

2. Apply load P_y at $z = \pi / 4\beta$

$$M = M_A F_1 + R_A F_2 / (2\beta) - y_A \, 2 \, EI \, \beta^2 F_3 - \theta_A \, EI \, \beta \, F_4 + LT_M$$

$M_A = 0$
$R_A = 0$
$y_A = -P_y A_1 / (EI \beta^3)$
$\theta_A = -P_y A_2 / (EI \beta^2)$
$LT_M = -P_y F_{a2} / 2 \beta$

$$M = 2 P_y A_1 F_3 / \beta + P_y A_2 F_4 / \beta - P_y F_{a2} / 2 \beta$$

$\beta z = \pi / 4$

$\text{Cos } \beta z = \text{Sin } \beta z$
$= 0.7071$

$\text{Sinh } \beta z = 0.86867$
$\text{Cosh } \beta z = 1.32461$

$A_1 F_3 = (0.5 e^{-\beta z} \text{Cos } \beta z)(\text{Sinh } \beta z * \text{Sin } \beta z)$
$= 0.04514$

$A_2 F_4 = [0.5 e^{-\beta z} (\text{Sin } \beta z - \text{Cos } \beta z)](\text{Cosh } \beta z * \text{Sin } \beta z - \text{Sinh } \beta z * \text{Cos } \beta z)$
$= 0$
$F_{a2} = 0$

$\alpha = 2 * 0.04514$
$= 0.09028$

$M = 0.09028\,(P_y / \beta)$

This condition relates to wheel load located near girder ends when (**$\beta z \le 3$**)

Similar to the results from case (a) for "beams of infinite length," the maximum moment occurs at **$z = 0$** but attenuates to a negative value near **$z = \pi / 4\beta$**. The difference is that this is a negative moment comes with a much **higher intensity** ($\alpha = -$ **0.3324** versus + **0.25**) giving a reversal sum **$\alpha_{range} = 0.5824$**

Interestingly when the wheel travels within a short zone spanning from **$z = 0$** to **$z = \pi / 2\beta$**, the extreme fiber in the top flange near support points of girder can experience a longitudinal stress reversal from behaviors applicable to both "beams of infinite length" and "beams of semi-infinite length" with a range of:

$\alpha_{range} = -(-0.3324) + 0.09028$
$= 0.42268$ **reversal sum**

Given in the example, the maximum reversal sum **$\alpha_{range} = 0.5824$** could be quite substantial (for not only the girder top flange but also the rail)

Based on the moment of inertia for the 100# rail $\mathbf{I_r} = 44$ and that for the top flange $\mathbf{I_{ef}} = 24$; $\mathbf{I_{xx}} = 68$, the flange local bending stress could be calculated:

$f_{bfc} = [\alpha\, P_y\, (2")\, (2 * 68 * 160)^{0.25} / (2 * 68)]$
$= 0.1786\, \alpha\, P_y$
$= 0.04465\, P_y$ for $\alpha = 1/4$
$= 2.2325^{\,ksi}$ for $P_y = 50^{\,kip}$

7.7 Top Flange Local Bending Stress – Potential Problem

The resulting girder flange stress $\mathbf{f_{bfc}} = 2.2325^{\,ksi}$ is the straight ± value representing either tensile or compressive in nature based on the condition if letting $\mathbf{P_y} = 50^{\,kip}$, but if only considering tensile stress fluctuation in particular then the **f_{bfc} stress range** would become:

(**0.5824** / 0.25) * 2.2325
= 2.3296 * 2.2325
= $5.2^{\,ksi}$

Obviously the $5.2^{\,ksi}$ could have consumed into the precious portion of the fatigue allowable stress range that might cause "big" problems (near a bolt hole or stitch weld) if this brand of stress does survive by that amount (with $P_y = 50^{\,kip}$)

But let's say we are designing a crane girder for unlimited on/off cycles of live load application, for its top flange already with a calculated **f_{bfc} stress range** of $5.2^{\,ksi}$, and then:

How about consider only the fluctuation range of this "localized" longitudinal stress – although the element under evaluation is under substantial compression but for time being what if discounting the effect from primary sources – and see what could possibly be the problems with top flange?

- At any clean base metal, for fatigue stress category **A**,

  $\mathbf{F_{TH}} = 24^{\,ksi}$
  But the net allowable fatigue stress range now becomes $24 - 5.2 = 18.8^{\,ksi}$

- For a built-up girder at its top flange where the web interfaces with full penetration weld, for fatigue stress category **B**,

  $\mathbf{F_{TH}} = 16^{\text{ksi}}$
  Now the net allowable fatigue stress range = 16 – 5.2 = 10.8 $^{\text{ksi}}$
  Similar limitation also applies to spliced girders of greater length, too (usually longer than 100 ft)

- At the bolt connection at rail clip, for fatigue stress category **B**,

  $\mathbf{F_{TH}} = 16^{\text{ksi}}$
  The net allowable fatigue stress range = 16 – 5.2 = 10.8 $^{\text{ksi}}$

- If there is a stiffener under the flange then for fatigue stress category **C**,

  $\mathbf{F_{TH}} = 10^{\text{ksi}}$
  The net allowable fatigue stress range = 10 – 5.2 = 4.8 $^{\text{ksi}}$

- If there is a wear plate stitch welded to the top flange, for fatigue stress category **E**,

  $\mathbf{F_{TH}} = 4.5^{\text{ksi}}$
  The **net allowable fatigue stress range becomes negative – meaningless in practical sense, or there is a potential fatigue issue looming**

Enough of numerical drill already, what demonstrated above was a number of scenarios from playing with a constant local stress $\mathbf{f_{bfc}}$ of 5.2 $^{\text{ksi}}$ alone against various fatigue stress categories just to prove a point as we said without even considering the "primary stress" from other "primary sources".

In this case there might not be a problem hopefully the compressive stress from primary sources is always greater than ½ of f_{bfc} *just as calculated; but what about material yielding, compressive strength qualification per non-fatigue mandate?* Also when dealing with unsymmetrical sectioned members, what if the elastic centroid is far apart from the shear center?

The rest of the story should be pretty much self-explanatory especially when there indeed is a top-flange hot spot connection detail fitting as being fatigue stress category E

In this case one should really watch out for the (undisputable) consequence (of cracked base metal) if the maximum calculated stress range including effects from primary stress sum – from flexure bending due to ±Px, ±Pz and Py and warping normal stress $\pm\sigma_n$ − does exceed the net allowable fatigue stress range

Although by calculation, there might not be any tensile fluctuation in top flange but the flange is not off the hook completely; for which it can be totally clear of fatigue failure only when the shear stress reversal does not exceed the allowed $\mathbf{F_{TH}}$.

The fact is; the local bending stress in the top flange is "relatively" not so significant, some of the Codes or Design Guides published in different era(s) may or may not recognize or mandate the incorporation of local bending stress of current concern into design consideration; it is then the designers' judgment to "see" for themselves in those situations whether if such local effect could be ignored or should be included in the **CRG** design.

For instance:

For symmetrical sectioned girders if intuited by flexure influence based on simple **M c / I**, one may take exception (or take chance from that) and do nothing about $\mathbf{f_{bfc}}$ based on the *assumption* (or the fact?) that due to the relative heavy $\mathbf{P_y}$ vertical load alone, the girder top flange is almost always under tremendous amount of compression from the primary stresses, but in reality one still needs to make sure;

Is that really true?
Has it been justified with calculation?
What about adding the effect (+ torsion) from lateral load $\pm\mathbf{P_x}$?
Or what about the effect from rail misalignment (+ torsion)?
What is the fatigue stress category as detailed?
What is the shear stress reversal effect? …

In summary, there is no such thing as trusting full-heartedly the influence only from simple **M c / I**

What was established with focus in **CRG** top flange could even be more interesting to the **100#** rail as we would extrapolate the example a bit further;

Given the rail section modulus at rail head $\mathbf{S_{HEAD}}$ = 14.6, the bending stress in which would be:

$$f_{rail\_head} = \pm [\alpha P_y (44 / 68) (2 * 68 * 160)^{0.25} / 14.6]$$
$$= \pm 0.135 P_y \qquad \text{for } \alpha = 1/4$$
$$= \pm 6.728^{ksi} \qquad \text{for } P_y = 50^{kip}$$

It is true that both the local bending stresses $\mathbf{f_{bfc}}$ in top flange and $\mathbf{f_{rail\_head}}$ in rail due to vertical load $\mathbf{P_y}$ were in measure of relatively small quantities compared to the stress due to primary load effects, but, it could be tricky if not risky to downplay the seemingly negligible amount by saying “What is the big deal?”

Then perhaps we should look at $\mathbf{f_{bfc}}$ and $\mathbf{f_{rail\_head}}$ subsisted in each subject entity with a different frame of mind:

- The $\mathbf{f_{bfc}}$ stress in top flange is rightfully so distributed as being truly localized considering the fact that flange element is not the entirety of the girder but is only one of the many components of the full girder cross section, therefore $\mathbf{f_{bfc}}$ stress should not spill over onto other portions of the girder section

  One should simply let $\mathbf{f_{bfc}}$ stress take its own course in joining the family involving longitudinal stress of other varieties (primary strong-axis bending, primary weak-axis bending and primary warping) within the top flange’s confine as part of the full-blown structural engineering qualification process

  One last important point to consider is that the $\boldsymbol{f_{bfc}}$ *stress calculation should be based on the* ***effective flange section****, not on the gross section.* This is one of the curve balls thrown at those who insist on using finite element method for everything

- The stress subsists in the railhead as $\mathbf{f_{rail\_head}}$ can also disburse down over the full rail profile. With calculated value of $\pm 6.728^{ksi}$ for $P_y = 50^{kip}$ as demonstrated, one would have doubled it to 13.456^{ksi} when dealing with fiber stress fluctuation involving tension

  Giving concern over rail’s wellbeing may be out of place under structural discipline. But for sake of longevity of all disciplines not only for the rail and the rail clips but also for superior performance of mechanical services, one may need to take $\mathbf{f_{rail\_head}}$ seriously in the evaluation for rail fatigue considering the influences from $\mathbf{P_x}$ and $\mathbf{P_z}$.

Normally the subject would have come to a conclusion; but wouldn't we at all have any hesitation with the girder web, which had been idealized as our equivalent **Euler** column?

<u>First of all – there could be a (potential) problem</u>:

We did start out with a theoretical value of $\mathbf{k_e}$ based on a *fixed-fixed* member = 0.5 so that we arrived at the flange local bending stress as:

$$f_{bfc\_0.5} = \alpha\, P_y\, t_f\, (4\, k_e\, I_{xx}\, h_w / t_w)^{0.25} / (2\, I_{xx})$$
$$= \alpha\, P_y\, t_f\, (2\, I_{xx}\, h_w / t_w)^{0.25} / (2\, I_{xx})$$

That was carried out herein short of any Official **R&D** endorsement in this context (as of this writing) yet

Although not an oversight but on the assumption that the flanges could hold out for taking $\mathbf{K_e}$ as 0.5; on the other hand to some naysayers, this could be arguably optimistic over the effective length if we opted for a recommended $\mathbf{k_e}$ value of 0.65 (per **AISC Effective Length Factor Table C-A-7.1**. as of this writing) and thus there would be $(0.65/0.5)^{0.25} = 1.0678$ increase in stress:

$$f_{bfc\_0.65} = \alpha\, P_y\, t_f\, (2.6\, I_{xx}\, h_w / t_w)^{0.25} / (2\, I_{xx})$$
$$= 1.0678\, f_{bfc\_0.5}$$

<u>Then there's another problem</u>:

We made an assumption upfront, in order for a localized web segment to function as if both ends were fixed; the girder must be prevented from being sidesway-buckled during all loading stages.

However, we all understood that crane runway girders are loaded at the top of the rail such that torsion becomes inevitable because:

- If there is rail offset against the **shear center** and/or
- Whenever lateral loads do participate in the action

What happens when torsion joins in the action?

The girder would rotate about the longitudinal axis through **shear center** − not to mention that even without any acquired torsion but for unsymmetrical sectioned member it tends to rotate on its own dead weight at any rate − and the fact that under torsion's influence:

- Along **x**-axis, the top flange would move one way and the bottom flange would move the other way

- **AISC J10.4** (as of this writing) on Web Sidesway Buckling stated "*This section applies only to compressive single-concentrated forces applied to members where relative lateral movement between the loaded compression flange and the tension flange is not restrained at the point of application of the concentrated force*"

Suddenly we need to take care of Web Sidesway Buckling, which is a global stability design issue. For this very reason, the relative movement between the two flanges during the active loading session would have invalidated the $\mathbf{K_e}$ value be it 0.5 or 0.65, agreed?

Therefore as minimum, this phenomenon should fit in as depicted in **AISC** Effective Length Table case (c) that has a recommended $\mathbf{K_e} = 1.2$ so then:

$$f_{bfc\_1.2} = \alpha\, P_y\, t_f\, (4 * 1.2\, I_{xx}\, h_w / t_w)^{0.25} / (2\, I_{xx})$$

$= 1.2447\ f_{bfc\_0.5}$

As a maximum, conservatively, if we were going further with Effective Length Table case (e) that recommended $\mathbf{K_e} = 2.1$ then:

$f_{bfc\_2.1} = \alpha\ P_y\ t_f\ (4 * 2.1\ I_{xx}\ h_w / t_w)^{0.25} / (2\ I_{xx})$
$= 1.4316\ f_{bfc\_0.5}$

Finally for $\mathbf{P_y} = 50$ ^{kip}, the **Tensile Stress Fluctuation** (**TSF**) based on $\mathbf{K_e} = 0.5$ or $\mathbf{f_{bfc\_0.5}}$ had been calculated as $\mathbf{TSF_{0.5}} = 5.2$ ^{ksi}, then by direct ratio with respect to $\mathbf{TSF_{0.5}}$ we obtained:

$TSF_{1.2} = 1.2447 * 5.2 = 6.472$ ^{ksi}
…

$TSF_{2.1} = 1.4316 * 5.2 = 7.444$ ^{ksi}

<u>A bit of overstated set back here but just to make a point while not considering effect from global primary load source:</u>

For any connection detail(s) classified as fatigue stress category **E**, the gross $\mathbf{F_{TH}} = 4.5$ ^{ksi}, but the net allowable fatigue stress range after deducting the calculated $\mathbf{TSF_{2.1}}$ from that would **remain being negative**

<u>Or on paper (on calculation pad) in a way, the connection may have serious fatigue trouble</u>

Even for fatigue stress category **C**, $\mathbf{F_{TH}} = 10$ ^{ksi}, the net allowable fatigue stress range becomes 10 - 7.444 = 2.556 ^{ksi} or it could become **negative** if $\mathbf{P_y} = 70$ ^{kip}. Or however we play the number game, quite likely that the threat could also apply to fatigue stress category **B**

After all, the rescue can only come from the compressive stress from global primary load source, "hopefully" it brings in a net compression; yet don't be too optimistic by such "hope" until confirmed by calculation. However, when in doubt or otherwise, we could see a clearer picture after delivering answer to a number of questions that one may ask:

- Should we consider top flange local bending?
- If we should then, does the top flange interact with rail as layered beams?
- If true then, does it fit as equivalent "**Beams on Elastic Foundation**"?
- If true then, should we consider $\mathbf{K_e} = 0.5$, 0.65, or $\mathbf{K_e} = 2.1$ instead?
- Is there any net tensile stress fluctuation, or shear stress reversal?
- What about the rail?
- …

So this subject is as debatable as we see it here clearly. The fact is, even if the Design Code or Design Guide did not make specific reference to "top flange local bending stress" doesn't mean that such stress does not exist.

7.8 Permanent Warps or Tilts – The Phenomenon

While making assessment of in-service performance of a typical Crane Runway Girder's with focus on structural serviceability, the least welcoming news to concerned <u>Engineers</u>, <u>Crane Operators</u> and <u>Mill Maintenance crews</u> and the like should be the sighting of "major" irregularity in the deformed geometry exhibiting unwarranted bent out of shape, sweeps, warps, tilts or twisting, etc.

The word "major" under this context signifies a condition that these deformations are not of temporary nature

In order to make better engineering sense, geometric imperfections such as *warps, tilts, or twisting* should have been resolved or charged against **shear center** rather than other nodes of interest or non-principal axes of reference.

Some of the imperfections are much more apparent to observers than from other varieties whether that came to light in official inspection reports or not

These issues or imperfections, for once befalling a permanent state, would either remain (active) as is or become more acute if there isn't corrective action taken. Imagine when this happens, how can the rail stay being flat, and what could happen to the end trucks?

And should anyone intended to relieve a recorded imperfection off the "action item list" then it should be properly evaluated to confirm at what level the damages had already been done to the structure first then take proper action

If a runway girder were "**qualified**" to take on imminent torsion under normal load applications, then by normal expectation any structural responses or deformations thereof should stay under elastic limit being a temporary phenomenon, **or else** permanent warps and/or twists would develop − and it won't matter whether if the structures were newly installed or long after being placed in services.

Yet permanent warps imparted to **CRG** on the mild side, even if not seriously enough as to inducing physical damages or causing crane operating issues − for instance, tendency to jump track − could be a telltale indication of *innate structural weakness against torsion and/or flexure as well*, Such a fact no one should deny.

Furthermore, "things" or "anything" could happen to any girder having unsymmetrical sectioned profile if it were "**not properly designed**" especially the longer-spanned ones:

- The member is prone to rotate about the **shear center** even though there is no live load applied – all because the **shear center** and **elastic centroid** are "offset" from each other

  Herein for one more time to the disbelievers: **Elastic centroid** in other word is the **center of gravity** where the integral mass of girder profile would pass through it without inducing rotation so long as the mass resultant passes through both **elastic center** and **shear center**; but since **elastic centroid** is at "offset" from **shear center**, the non-zero **x**-projection of the "offset" dimension becomes the torque moment arm

  For symmetrical sections, both **elastic centroid** and **shear center** dwell on the same axis of symmetry thus the **x**-projection is zero but it is non zero for unsymmetrical sections thus the girder section has to rotate about the **shear center**

- If the girder was "**not properly designed**" then the perceptible deformation under live load applications would be more than being moderate; so if the ensuing "appearance" of which had already caught the attention of Inspectors or Field Engineers then, the same "look" could confer a more cynical impression as if the member has been *deformed beyond elastic stage* − a reasonable supposition to most observers

But generally speaking, be that an exaggerated visual effect to eyes of the beholders or an incurred inconvenience to the crane operators in service, "untreated or mistreated" warping or twisting phenomenon is in fact more severe than the allowable tilts or misalignments if so as limited per design criteria – provided the mandate was clearly specified and reasonable − which is supposedly much more tolerable from a design standpoint.

The warping (actually the tilting) in our focus features permanent displacement of:

- The top flange moved into one lateral direction and
- The bottom flange moved into the opposite direction

Due to the occurrence as top flange and bottom flange had displaced sideway along opposite **X**-directions, an angular rotation **θ** about the **CRG** longitudinal **Z**-axis passing through the **shear center** would take shape as a result

The interest at his point is quite simple,

Not to go after (1) the reason why that happened, or (2) ways how to prevent such situation from happening, or (3) the fact would it lead to side-sway buckling or other stability issues, etc.

But through practical means to examine what is the prescribed stress level owing to stress-strain compatibility knowing that the permanent deformation has already taken place

Θ of interest is the angular measure calculated based on the *numerical division of simple (vector) sum of respective displacement at two flanges by the distance in between the flanges*.

Any quantity of **θ** (by the Book) is merely the geometric measurement of acquired rotation about **shear center**; to us herein it could be viewed as the angular imperfection. But regardless how we think, **θ** does not induce stresses in the **CRG** except for the trouble that it could become a major serviceability issue much sooner if not later when the crane railhead could no longer "lay flat" owing to the tilting if conditions worsened.

7.9 Permanent Warps or Tilts – The Implication

Let symbols **θ'**, **θ''**, and **θ'''** to correspond respectively with the 1st, 2nd and 3rd derivatives of rotation **θ** through the **shear center**, all of which could be obtained once the **θ** function has been defined. Per classic torsion theory applicable to open sections, **θ'**, **θ''**, and **θ'''** are the sources of ***St. Venant*** *torsional shear* τ_0, *warping normal stress* σ_n *and warping shear stress* τ, respectively.

A simplified treatment to these stresses could be illustrated using a simple example.

Example 7.2

Given:

A girder of 100 ^{ft} long having an out-to-out depth of 132",
Flexural and torsionally simple supported at both ends,

On measurement using a common reference **y**-axis as a shared base of positive or negative pointing sense along **x**,

The field survey data taken at 5 discrete stations located at **z** = 0', 25', 50', 75' and 100' from the left support recorded respective the lateral translations:

x_{top} = 0.5", 0.75", 1.1875", 1" and 0.75" at the top flange,
x_{bot} = 0.25", –1", –1.375", –0.25" and 1" at the bottom flange

Required: Evaluation of the torsional effects due to the as-measured lateral translations

Solution:

Survey data indicated that the girder had been twisted/warped due to translation of the top flange and the bottom flange along opposite directions

Prior to applying mathematical formulas for evaluating torsional effects related to twisting angle $\boldsymbol{\theta}$ and/or to its successive derivatives, the first step is to develop the algebraic equation representing (approximating) the $\boldsymbol{\theta}$ about cross section's **shear center**

Define these parameters/notations:

L = Girder length = 100'
d = Girder depth = 138"
t = Component thickness
G = Shear modulus
E = Young's modulus
ω_n = Nodal unit warping
S_w = Nodal warping static moment
Z = Coordinate in ft measured from left support

ε_t = Translation at top flange
ε_b = Translation at bottom flange

In order to facilitate calculation of $\boldsymbol{\theta}$ corresponding to any arbitrary **Z** within the span, algebraic functions for both $\boldsymbol{\varepsilon_t}$ and $\boldsymbol{\varepsilon_b}$ must be established beforehand

This may be accomplished by non-linear curve fitting using polynomial regression for which the function usually consists of multiple terms in power of **Z**

Depending on the chosen order of terms and the data range requirement, the *top flange curve fitting* its five points may be taking after any one of these equations taking the form:

$\varepsilon_t = a_0 + a_1 Z + a_2 Z^2$
$\varepsilon_t = a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3$
$\varepsilon_t = a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3 + a_4 Z^4$
$\varepsilon_t = a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3 + a_4 Z^4 + a_5 Z^5$
$\varepsilon_t = a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3 + a_4 Z^4 + a_5 Z^5 + \ldots$

Where {$\mathbf{a_0}$ through $\mathbf{a_5}$ …} are the unknown coefficients yet to be determined. Normally the number of unknowns would match the number of data points but could also be different than that (through numerical coincidence?)

Since five data points were given, try the expression as follow:

$$\boldsymbol{\varepsilon_t = a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3 + a_4 Z^4}$$

Choose not more than five and not less than five and see − Matching exactly five unknowns with five preset data points yields an optimum function as to formulating a continuous curve

> Yet by conventional wisdom or otherwise, it may confer at this point a false impression that terminating the function at term $\mathbf{a_4 Z^4}$ could introduce inaccuracy through truncation errors from omitting other higher ordered terms
>
> But that idea is somewhat misleading in this case since truncation errors are in fact more common in computing approximated value representing the expansion of an infinite series whereas the dealings here are for the discrete data pairs obtained through measurement by survey instrument

The graph yet to be plotted − based on data points with accuracy dictated by data given by the survey log − should conform mathematically to a single curvature as expected; but if ever there were any inherent errors then most likely it would have come from the recording or reading of the survey results

With the pairing of survey data at the top flange:

$\varepsilon_t = 0.5$ at Z = 0
$\varepsilon_t = 0.75$ at Z = 25
$\varepsilon_t = 1.1875$ at Z = 50
$\varepsilon_t = 1$ at Z = 75, and
$\varepsilon_t = 0.75$ at Z = 100

The next task is obtaining five simultaneous algebraic equations to be solved for the coefficients $\mathbf{a_0}$ through $\mathbf{a_4}$

Before proceeding further, take note of a practical situation in that for girders of much greater length with sparsely spaced data points, the calculated native magnitude of $\mathbf{Z^4}$ with have been 100,000,000 for **Z** = 100 that can become too large when compared with all the other as-measured $\boldsymbol{\varepsilon_t}$ so *then what?*

*To avoid probable numerical **overflow** and **underflow** situation out of handling large numbers mixing with small numbers simultaneously, it helps to **normalize** the coordinate **Z** by a common factor*

In this example because the survey stations were given at "even" 25 ^{ft} spacing, conveniently one could and should, but not necessarily, use the same matching quantity (as the factor) for normalization

There is no such thing as absolute right factor or absolute wrong factor for any specific task, and it all depends on the application and the dimensional units chosen, or on personal preference or on numerical performance, etc.

After applying the factor of 25 to **Z** then:

z = Z / 25

Noticing from the above relationship that,

In pure numerical sense, there implanted a considerable parametric difference between the upper case "**Z**" and the lower case "**z**" here; the bottom line meaning: **z** is now a normalized variable

Following that, one could rewrite the top flange curve equation using appropriate symbols as:

$\varepsilon_t = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4$

Start solving from z = 0, $\varepsilon_t = 0.5$
Therefore $a_0 = 0.5$

Next substituting:

$\varepsilon_t = 0.75$ for z = 25/25 = 1
$\varepsilon_t = 1.1875$ for z = 50/25 = 2
$\varepsilon_t = 1$ for z = 75/25 = 3 and

$\varepsilon_t = 0.75$ for $z = 100/25 = 4$

One obtains the equation set:

$$
\begin{aligned}
a_1 + a_2 + a_3 + a_4 &= 0.75 - 0.5 \\
2\,a_1 + 4\,a_2 + 8\,a_3 + 16\,a_4 &= 1.1875 - 0.5 \\
3\,a_1 + 9\,a_2 + 27\,a_3 + 81\,a_4 &= 1 - 0.5 \\
4\,a_1 + 16\,a_2 + 64\,a_3 + 256\,a_4 &= 0.75 - 0.5
\end{aligned}
$$

Solving for all unknowns:

$a_1 = -0.4583$
$a_2 = 1.1302$
$a_3 = -0.4792$ and
$a_4 = 0.05729$

After applying the same procedure for the bottom flange data, we arrive at:

$z = Z / 25$ and

$$\varepsilon_t = 0.5 - 0.4583\,z + 1.1302\,z^2 - 0.4792\,z^3 + 0.05729\,z^4$$
$$\varepsilon_b = 0.25 - 0.9792\,z - 0.7917\,z^2 + 0.6042\,z^3 - 0.0833\,z^4$$

Prior to moving further it is imperative to verify the derivations:

- Calculate separate set of ε_t and ε_b by the last two equations at closer **Z** intervals, say at every 5 feet or every 12" and

- Plot the curves then inspect visually for any unusual results

When all is done well:

$d = 138"$

$$\theta = (\varepsilon_t - \varepsilon_b) / d$$
$$\theta = 0.0018116 + 0.003775\,z + 0.01393\,z^2 - 0.007851\,z^3 + 0.001019\,z^4$$

$$\theta' = d\theta / dz$$
$$\theta' = 0.003775 + 0.02786\,z - 0.02355\,z^2 + 0.004075\,z^3$$

<u>Important − remember to reapply the normalization length Z for the actual value of θ'</u>

τ_0 = St. Venant torsional shear stress
$= t\,G\,\theta' / (25' * 12")$
$= 0.003775\,t\,G / 300$ (maximum at z = 0)
$= 0.1404\,t$

$$\theta'' = d^2\theta / dz^2$$
$$\theta'' = 0.02786 - 0.047106\,z + 0.012225\,z^2$$

σ_n = Warping normal stress
$= (E\,\omega_n\,\theta'') / (25 * 12)^2$
$= 0.007363\,E\,\omega_n / 90000$ (maximum at z at mid-span)
$= 0.002373\,\omega_n$

$$\theta''' = d^3\theta / dz^3$$

$\theta''' = -0.047106 + 0.02445\ z$

τ = Warping normal shear stress
= $(E\ S_w\ \theta''')\ /\ t\ /\ (25 * 12)^3$
= $0.047106\ E\ S_w\ /\ t\ /\ 27000000$ (maximum at z = 0)
= $0.0000506\ S_w\ /\ t$

Stresses $\boldsymbol{\tau_0}$, $\boldsymbol{\sigma_n}$, and $\boldsymbol{\tau}$ at any girder component node at any coordinate **z** could be evaluated once the corresponding value of **t**, $\boldsymbol{\omega_n}$ and $\mathbf{S_w}$ are known

But just so most importantly for unsymmetrical sectioned members both $\boldsymbol{\omega_n}$ and $\mathbf{S_w}$ are highly dependent on *the properly located **shear center** and the correctly calculate **warping constant***.

Anyhow, the girder is obviously in very serious distress for sure after it had undergone plastic deformation to this magnitude. This is a classic example of what can happen when the girder is extremely weak in resisting torsion.

*Unfortunately, as there is no official design document of this girder for review, but to take a guess, for sure the designer must have assumed that the **shear center** is located at the rail base like many engineers do. It ended up the torque moment arm is 6" at best – regardless to the girder profile being symmetrical or unsymmetrical – so how can that make any sense compared to **the true shear center location** of a 138" deep girder is the question.*

7.10 Variable Sectioned Members – Atypical but Not To Be Ignored

Unless for special reasons otherwise, the profile **geometry** of typical **CRG** in most applications stays the same in all **X/Y** aspects from one end to the other, *except for those detailing-related dissimilarities inherent within local confine that must adapt to presence of welds and bolts or mechanical attachments*, etc.

On verifying consistency in geometric attributes,

One could take multiple **X/Y** profile slices along the span *from **Z** = 0 to **Z** = L* and should be able to verify and acknowledge whether the "span-wide" **uniformity** remains true and valid or not

Span-wide Uniformity is valid provided for each and every profile slice (1) the in-plane *geometric contour dimensions*, (2) the relative positioning and elementary layout *arrangement of components* and (3) the *thickness/length/width of* respective *component do not vary*

On geometry-modeling intent,

Once we confirmed the "span-wide" geometric **uniformity** is valid – which is an ideal state of geometric **Z-independency** − we could appreciate that a single set of **X/Y** node/element incidence scheme "could be" served as a "benchmark profile input suite" that is supposedly applicable to the entire girder

Remember, this **Uniformity** only settles the geometric aspect; it has nothing to do with where actually the bolts or the welds are located yet, that in other words we meant *physical connection topology*

The establishment of "benchmark **X/Y** node/element incidence specification" is meant for all **Z**-profile slices to be based on for geometric input purposes:

- Whether for individual slice as standalone entity, or for
- Several adjacent slices being grouped into subdivided girder Zone(s) and whatnot

Therefore the very same collection of **X/Y** nodes would apply **at every Z-coordinate;**

> Provided that, once again, the given member from one end to the other has *uniform section outline, nodal link segment arrangement and is identical in all other component detailing aspect*

But is that realistic?

So far so good it seemed, but there is a catch:

On principle, the benchmark profile idea works as if limited for depicting "basic" section outline geometry only, and no more − but that may or may not be true (depending on how we can take advantage of the setup)

In theory, as soon as certain attribute(s) of constituent element changes from one **Z**-slice to the next **Z**-slice − such as variation (1) in element's depth or thickness or (2) in component's detailing features, etc. − as result, the supremacy of benchmark profile geometry although may still be in effect, but sure deserve an expanded review on how to take in those "smaller-scoped variations" into consideration and at the same time maintain focus on the bigger picture

For special reasons,

Should the given girder geometry were of variable sectioned such as the one illustrated in **Figure 7.1** below − *with variations (1) apparent in the depth, (2) not so apparent in components' width or thickness, etc.* − then appropriately, the input data domain must reflect the disparity passing through from one slice to next slice or from one Zone to next Zone that calls for multitude of **X/Y** node/element specifications

What needed is a strategy on how to modify the benchmark profile that would adapt to differences not only in the *geometric contour* but also in *physical connection topology*

It is imperative to pay attention to and follow through (1) with the "Girder mid-span/Axis of symmetry" prescribed in **Figure 7.1** that pretty much sets the limitation of application scope and (2) on personal deduction into what works and what doesn't, and (3) on whether if the "opinion appeared further into this Chapter" applies or not.

Take a fresh look at a "generalized" variable-sectioned girder with simply supported boundary condition at both ends in both flexural and torsional senses;

The complexity inherent in handling of the given structure, yet to be reflected from what comes next, would depend on (1) if it's symmetrical or unsymmetrical sectioned, (2) how is the member being loaded with respect to **shear center**, and (3) what is the limit of our engineering goal based on what given

Nonetheless, among all the must-do routines in treating variable sectioned members, some of the very elementary chores can still be mastered handily by *simple static force equilibrium* for carrying out common tasks such as:

- Analyzing the structure for *global* structural responses – if only we were to scope out the support reactions, flexural shear diagram or bending moment diagram, etc.

- Calculating flexural bending stress and flexural shear stress, etc.

All seemed to fit in as a clear-cut top-down process.

> So once the calculation of basic flexural load responses and flexure-related stresses are over with and that should be the end of the process as far as *simple static force equilibrium* goes, which is far from what needed to accomplish a much generalized full-blown **CRG** structural qualification

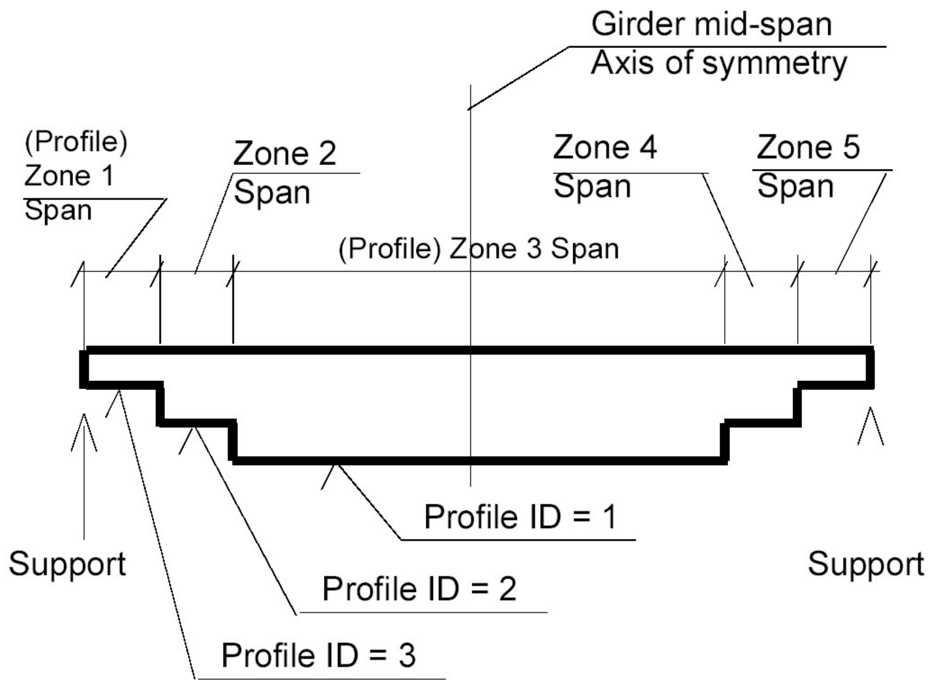


Figure 7.1 Variable Sectioned Member

To have a feel for, just give a "serious" thought on how to deal with these subjects of interest:

- Easily we had calculated bending moment and flexure shear and the corresponding flexure-based stresses, but how can we calculate **shear-center** based load responses and torsion-related stresses?

- How to calculate the maximum deflections at railhead combining **X/Y** rigid-body movement and **Z**-rotation about **shear center**, and then verify the results making sure that there is no (potential) serviceability issue?

- What form of data cache setup is needed?

Predictably though, extra efforts are needed to investigate structural behavior of variable sectioned members and what it takes to do it all would be increasingly challenging once we stepped *beyond the limit of simple static force equilibrium* – that is universally true for **CRG** and **non-CRG** structures.

Before going further, one needs to understand the difference between geometric *uniformity* and *regularity*, in particular among variations of cross-sectional configuration being brought up herein:

Definition of *uniform section* and *variable section* in general are **Z**-based while the specifics in *symmetrical section* and *unsymmetrical section* are **X/Y**-based

What makes it so challenging is how to qualify the adequacy of **CRG** structure with configuration combining unsymmetrical section and variable section into one entity as depicted in **Figure 7.1**.

Having knowledge of what to do or how to do is not enough, but how to maintain the level of accuracy of result obtained from whichever methodology adopted becomes the key.

On meeting the challenge,

No matter (1) how elegant the ways and means were organized from the starting point, (2) how accurate the results realized at the finish, and (3) regardless to the process or procedure that were flawed or not, one does not have to go very far to realize that numerical woes take hold right away when we *start messing with section properties*

Remember garbage-in garbage-out;

In solving specific structural problems, no other concerns could be more crucial than (1) calculating correctly the value of section properties and the apply loads, (2) analyzing the structure using authentic approaches and (3) maintaining the accuracy of results all the way till process ends

A careless slip during any one of those steps would definitely fail the exercise at somewhere, and that somehow would affect either the structural response values at *analytical stage* or the internal stress follow-up from these responses at *design stage*, etc.

That said is universally true for members of any configuration. Variable sectioned members, as much as to be avoided for practical purpose if possible, are atypical as they were but not to be ignored. The burden to Structural Engineers is how to maintain the consistent engineering objective while engaging **Crane Runway Girders** inherent with variable cross section geometry.

7.11 Variable Sectioned Member – Limitation of Classic Treatment

Take the *generalized cross section profile geometry* subsists in any structural member; no matter (1) how random (or how consistent) the variations along the member length that were or (2) how impractical (or how practical) the problem definitions there as given, and yet:

How to "reasonably" predict and "properly" handle the *generalized* structural behavior for structure given *generalized* profile configuration under *generalized* loads now become the most *generalized* point of issue

The engineering norm when sticking with simple bending:

By way of simplified analytical functions, one could fall back on several "**classic treatments**" drawn from area-moment (or moment area) method, conjugate-beam or slope deflection method, etc. During the progression of each method as aforementioned, any numerical clutter as result from maintaining compatibility among *applied loads, material strength, section properties, support reactions and the boundary conditions and so forth* could all be resolved by proper formulations revolving around

- The flexural moment diagram and

- The magical "**EI**" bundle, for which the "**I**" value could remain as constant throughout or vary from one (**XY**) slice to the adjacent slice, or from zones to zones

These classic treatments should work out for simple applications but with limited scope

On given problems when *involving load resultants through shear center leading to simple bending behavior* to be solved manually, it is much easier for a structural member having **simply support at both ends** with focus on entities related to the flexural elastic curve (flexural bending moments, flexural shears, slopes and deflections, etc.)

Otherwise although it can still be worked out for other types of support but it would become much more complicated

In other words, with exception to "indeterminate structures" and/or for interest further than dealing with global "deflection or rotation," it should make no difference for a structural member having either variable section or uniform section for meeting simple design goals with simple interests in response to handling simple applications

There is a catch:

At times the major source of engineering slip-up or disappointment could come from our misinterpretation in the significance contrasting between what is *generalized* and what is *simple*; not knowing the classic treatment strategy as remarked above works mainly for **non-CRG** structures under loading effects conforming to **simple bending** (beside other limitations)

Out of numerous **non-CRG** applications intended for various purposes involving variable sectioned members, perhaps web-tapered girders may stand out rivaling the complexity in section geometry variation of interest featured herein

And yet the technical advantage gained in the design of **non-CRG** web-tapered girders over the **"Disadvantaged Crane Runway Girders"** in general is, the former have been blessed by matured **R&D** endorsement in defense against flexural failures, especially the shear failure, resulting from "simple bending" due to loads passing through **shear center** − without inducing torsion, that is − otherwise it could be technically risky or it won't work at all

The point in the making is that *what works for **non-CRG** structures per latest **R&D** endorsement does not at all work for **CRG** with variable section* for practical reasons, considering loaded under these conditions:

- Moving loads
- Probable local buckling issue
- Reversible loads
- Load offset from **shear center**

One of the challenges most Engineers face:

There seems no holistic treatment (theoretical or practical, classic or modern) applicable to **variable sectioned members subject to torsion**, for which conjugated-beam, column analogy or slope deflection method would be of absolutely no use or plain wrong if *misused*

Moreover, in order for any mathematical formulation or engineering-mechanics treatment to be genuine for torsion relevancy, it must bring about expressions in terms of no less than **E, I, A, Q, J, G,** $\mathbf{C_w}$**, and most importantly −** $\boldsymbol{\beta}$**L**, etc. otherwise anything short of involving those parameters would immediately disqualify the suitability of **CRG** as to staying above and beyond the rank of simple bending.

7.12 Variable Sectioned Member – Beyond Simple Bending

In various applications we dealt with for various purposes, one has to apply a variety of cross sectional properties as needed to accommodate various calculation steps while going through with various stages of structural evaluation process.

Of **CRG** importance whether viewing these section properties in an absolute sense or on special circumstance, be they primal, secondary or standalone and whether in singular or in clusters, the row call head count of which and their bundles could be in the scores if not more.

On the numerical handling effort in general:

> For variable sectioned members, it may be fairly easy to compute certain properties if only that engage simple numerical procedure − such as multiplying the $\mathbf{width_{-1}}$ to the $\mathbf{thickness_{-1}}$ to come up with cross-sectional area $\mathbf{A_1}$ and then $\mathbf{A_2}$, and so forth
>
> But of no surprise, the said application being as simple as illustrated is of fairly limited practical consequence if torsion-based sectional properties − other than **St. Venant** torsional constant − were brought into the more serious kind of computation

For variable sectioned members – there is a stark contrast to uniform sectioned member in that an one-of-its-kind unambiguous suite of properties would work for the entire member from end to end − the effort that takes to look after an entire set/suite of section properties (consisting $\mathbf{A, I_x, I_y, Q_x, Q_y, J, C_w}$, **and** $\boldsymbol{\beta}$, etc.) is a different story because the calculation for selected properties could be quite involved – but rest assured, one just need to do it one step at a time

Along the girder span, consider multiple section slices with coordinate set corresponds to the collection $\{\mathbf{z_1, z_2, z_3 \ldots z_n}\}$ matching preselected *stations'* numerical range from $\mathbf{z_1} = 0$ to $\mathbf{z_n} = \mathbf{L}$;

> What is in need is a universal handling procedure suitable for calculating each and every property class associated with the member. To each property class, once finished with application to variation from $\mathbf{z_1}$ **to** $\mathbf{z_n}$, it renders a respective collection of constituent entities corresponding to each *station's* coordinate varying from $\mathbf{z_1} = \mathbf{0}$ to $\mathbf{z_n} = \mathbf{L}$
>
> The plan to cover every detail seemed cumbersome if taking into account of these tricky but in fact not so tricky situations:

- Pick a representative cross-sectional area "**A**" and first moment about principal **X**-axis "$\mathbf{Q_x}$" as sample property entities to expand the scope of what we need;

  Resulting from the expanded scope,

  > One would obtain the collection $\{\mathbf{A}\} = \{\mathbf{A_1, A_2, A_3 \ldots A_n}\}$ and collection $\{\mathbf{Q_x}\} = \{\mathbf{Q_{x1}, Q_{x2}, Q_{x3} \ldots Q_{xn}}\}$ and so forth, in which if $\mathbf{n} = 10$ then it means cross section **A** needs to be calculated 10 times; likewise the first moment about the principle **X**-axis $\mathbf{Q_x}$ needs to be calculated 10 times as well, and so does warping constant $\mathbf{C_w}$ and so forth;

  Imagine ten plus ten plus ten … is not a problem once getting the correct *formulas* and the proper *algorithm* going

- Among all the reasons that make **Crane Runway Girder** more unique than regular **non-CRG** structures, one of which that may set off data management headache is: What to do for section component(s)/element(s) with aspect ratio far exceeded the threshold quotient enumerated in **AISC Table B4.1**? (although there is other strategy involved in dealing with post-buckling effective sections)

With automation already being the tool of the trade, certainly a large portion of manual calculation becomes the thing of the past. Therefore it is no longer an issue repeating the same numerical chore 10 times or ten thousand times. A few things to concentrate on is:

- What methodology to use and
- How to manage the **I/O** data

Finally in making the automated scheme work better all over and knowing what is needed to watch out for is not limited to how to conquer the numerical complexity but also how to work that out into a unified approach practical to all applications whether dealing with **CRG** or **non-CRG**.

7.13 Variable Sectioned Member – The Need of an Out-of-the-Box Approach

As to accurately making generalized *engineering* common sense with respect to variable sectioned member, each cross sectional property (**A, I_x, I_y, Q_x, Q_y, J, C_w, and β**, etc.) appearing in the *engineering* calculation must be explicitly identified with *properly calculated quantity* associated with *proper engineering syntax,* **otherwise it won't make any sense**.

Pick an instance of section property and look into the matter;

Depending on (1) the choice of data formats and the data presentation scheme and (2) the uniqueness in the assignment of native dimension – square inch, inch to the power of 4 or 6, or dimensionless scalar – being labeled that varies from one type of elementary property to another, or the cluster of which, it could get fairly messy when facing so many distinctive collections of section property, regardless to how innocent the engineering action(s) that had started off from

For that very reason,

Even if by using hardcore **R&D** brute force means (if there is such chosen means that serves the purpose,) it may appear highly promising to develop an exact mathematical expression or a group of representative analytical functions as surrogate as to mimicking the behavior of *variable* sectioned member under *generalized* load influence

But, thanks to the two keywords − *variable* and *generalized*, in connection with catering to so many different entities suiting every given occasion, it would be quite **cumbersome** and utterly **unrealistic** if indeed what we are about to accomplish is through our so-called viable means

Among many functional methods nevertheless,

One of the approaches that deserve paying attention to (or an out-of-the-box approach) as to simplifying the daunting task of dealing with the vast diversity of variable sectioned properties is to transform for each property type from its original collection of many pieces of <u>variable</u> data (varying along stations z_1, z_2, z_3, …, etc.) into a single <u>equivalent constant property</u>

As follow are the drawback and advantage from using the transformation explained in simple words:

The upcoming scheme might not seem that straightforward upfront, but in hindsight when it ends and being applied onward to analyzing the variable-sectioned structure, it should *always be much easier in the dealing with the transformed "**one**" than tackling the original set of "**many**"*

A forewarning:

Using equivalent constant property is not some equivalence to taking crafty shortcut as it seems, but rather a rational way of simplifying the complex task on hand

The modifier phrase "equivalent constant" being "operated" to whichever section property does signify that the "resolution" obtained from using "that particular constant" for problem solving would be (at best) an approximation, and hopefully close enough to the real thing

The relevance from the aftermath of this **simplification** (or **numerical transformation**) is meant to be applicable to the entire structure of interest in full. In a generalized sense as the variable sectioned member with arbitrarily configured section geometry was subjected to a set of generalized load of set pattern, the rationale is:

*Expectantly the **transformed structure** with an array of "equivalent properties" would exhibit global structural responses in close resemblance to that of the **original structure** having variable properties* – the complication is how to make that work

In summary, to have any relieve from the complication imparted, what is in need in the interim is an expanded solution scheme that can work out for **CRG** engineering and beyond:

Unlike classic methods (for instance; slope deflection or moment area, etc.) that are limited to engaging only the flexural moment of inertia for rudimentary solution, the generic necessity is a scheme showing how to step beyond flexural moment of inertia into **St. Venant** torsional constant, warping constant and flexural first moment, and so forth

The scheme being proposed later on should be suitable for any arbitrary set of **data model**, including those with discrete numerical-geometric variations and discontinuities. Among many methods being adopted in engineering approximation, those employed as to conforming the shape-function of specific trait of a known set of data into an "infinite series" should be the better choice.

The one in particular favor is **Fourier** series, of which the practical application is usually much simpler than what takes to fully comprehend the theory.

7.14 Variable Sectioned Member – Development of Equivalent Section Properties

Implementation of "transformation" or "simplification" from a rather complicated post into manageable situation now becomes the next focus.

Pick any one of the representative property members − it doesn't make any difference which one (cross sectional area-a or warping constant-c, for instance) − from the collection of section properties to demonstrate how the transformation is done:

Given a data domain { **c** } representing an assembly of all the z-profile **warping constants** for the entire girder, in which each and every c-value had been individually calculated − ahead of time − based on the section geometric attributes applicable at the respective z-profile, z-slice or z-station of interest

In this example,

"Evenly spaced" along the girder length are total "i**"** number of stations or "i" number of slices arranged in sequence from station #**1** to station #**i**, correspondently the domain element registered in sequence: $\mathbf{c_1, c_2, \dots c_i}$, in which $\mathbf{c_1}$ is the warping constant corresponds to coordinate $\mathbf{z_1}$, $\mathbf{c_2}$ corresponds to coordinate $\mathbf{z_2}$, thus $\mathbf{c_i}$ corresponds to coordinate $\mathbf{z_i}$, etc.

Notice the inkling of preaching for "evenly spaced" stations or not is not that essential; but doing it herein merely is for better understanding of the concept; yet if so defined this way upfront and during the actual practice as we will find out that, it should be helpful during the upcoming *numerical integration process* when all is said and done

At this point just starting, it doesn't matter if the values in the collection { **c** }were duplicated, identical, all different, all the same or all arbitrary. However, if:

- Letting **C(z)** to represent a function depicting the warping constant value at any **z**-coordinate between **0** and **L**, also

- Transitioning the arbitrary girder length **L** mathematically into an arbitrary sinusoidal wave period which in this case is symmetrical about the mid-span (**Z** = **L** / 2)

Then the function **C(z)** may be approximated using a **Fourier** Sine series expansion, namely:

$$C(z) = b_1 \sin(\pi z/L) + b_2 \sin(2\pi z/L) + b_3 \sin(3\pi z/L) + \dots + b_n \sin(n\pi z/L) + \dots$$

In the expression $\mathbf{b_1, b_2, \dots b_n \dots}$ are the unknown **Fourier** coefficients

Notice that each term in succession has the period of $\boldsymbol{\pi, 2\pi, 3\pi}$ … and therefore the first term and the last term in the series would equal to zero for **z** = **0** and **z** = **L**

With characteristics as such **C(z)** would be a half-ranged function valid on the interval $\mathbf{0 \le z \le L}$ (*instead of a full-ranged function on interval* $\boldsymbol{-L \le z \le +L}$) thereby on conditions as follow the function would be valid:

- Only if the coefficients $\mathbf{b_1, b_2, \dots b_n \dots}$ have been determined and
- Only if the Sine series converges

*It is important to recognize that the index "**i**" denotes the upper limit – total number of stations − of the (final numerical integration) station identification, which is **NOT** related to the index "**n**" appearing as one of the subscripts for the "**b**" coefficient*

Readily doing it one by one, the value of coefficient corresponding to each term in the series could be determined by integration of simple trigonometric functions.

Start by taking a generic term **sin(nπz/L)** out of the series and multiply it by **sin(mπz/L)** into a product **[sin(mπz/L) sin(nπz/L)]** in that both **m** and **n** are integers.

Now there could only be two possible outcomes from the function product relation:

Case 1: when $m \neq n$,

$$\sin(m\pi z/L) \sin(n\pi z/L) = ½ [\cos(m - n)\pi z/L - \cos(m + n)\pi z/L)]$$

When the expression on the right side is integrated from z = 0 to z = L;
Result = 0

Case 2: when m = n,

$\sin(m\pi z/L)\sin(n\pi z/L) = \sin^2(n\pi z/L)$
$= ½\,[1 - \cos(2n\pi z/L)]$

Upon integrated from z = 0 to z = L;
Result = L / 2

By what demonstrated as in Case 1, integration over the product of a pair of discrete functions resulting into **0**, therefore mathematically speaking the set of functions { **sin(nπz/L)** } acquires the so-called **orthogonality** property over the interval **0** < **z** < **L** for integer **n** = 1, 2, 3, …, etc.

It follows that, if only:

- Multiplying **sin(nπz/L)** to the function **C(z)** and to each term on the right hand side, and
- Integrating for the entire expression from **z** = 0 to **z** = L

Then it should lead to the following expression:

$$\int C(z)\sin(n\pi z/L)\,dz = b_1\int \sin(\pi z/L)\sin(n\pi z/L)\,dz + b_2\int \sin(2\pi z/L)\sin(n\pi z/L)\,dz + b_3\int \sin(3\pi z/L)\sin(n\pi z/L)\,dz + \ldots + b_n\int \sin(n\pi z/L)\sin(n\pi z/L)\,dz + \ldots + b_{n+j}\int \sin(2\pi z/L)\sin[(n+j)\pi z/L]\,dz + \ldots$$

Herein the **Fourier** coefficients $\mathbf{b_1, b_2, \ldots b_n \ldots}$ remain being unknowns;

However, after applying the orthogonality property such that every term on the right hand side would have to be **0** except for $\mathbf{b_n \int \sin(n\pi z/L)\sin(n\pi z/L)\,dz}$; and its integration result (per Case 2) would become ($\mathbf{b_n}$ **L** / 2) which must equal to the left hand side $\mathbf{\int C(z)\sin(n\pi z/L)\,dz}$

All that turns out to be the generic solution statement; for all ***Fourier*** *coefficients in the series terms if expressed as a set of* ***Fourier*** *coefficients*:

$$\{ b_n \} = \{ (2 / L) \int C(z)\sin(n\pi z/L)\,dz \}$$

In which:

- The range of integration is from z = 0 to z = L and
- n = 1, 2, 3, …

Theoretically, after sufficient number of unknown $\mathbf{b_n}$ were determined, values as summed up from the **Fourier** Sine series could be digitized against various **z**-coordinates along the girder length, which should closely resemble the graphics implication from { **c** } intended to represent the **original variable warping constant data collection**.

The "important concept" of transformation is to establish a numerical equivalence between the two series *as if we were dealing with* ***two independent CRGs***:

One of the two **CRGs** is already **given** with all the geometric attributes **identified** and all the section properties had already been **calculated**, out of which the variable sectioned warping constant properties are delimited in a data collection { **c** }

The other **CRG** is an equivalent member with **unknown warping constant** yet to be determined

For the equivalent **CRG** member, one can:

- Designate the data collection of unknowns as { **c'** }

- And let the **equivalent** warping constant = $\mathbf{C_{eq}}$

Parallel ***Fourier*** treatment could now be repeated to { **c'** } provided that (1) could be depicted by a function **C'(z)** and (2) could be approximated by a **Fourier** Sine series expansion with yet the unknown **Fourier** coefficients $\mathbf{b'_1, b'_2, \ldots b'_n \ldots}$

Then, owing to the uniform property values from end to end, the following two expressions should be valid:

$C'(z) = b'_1 \sin(\pi z/L) + b'_2 \sin(2\pi z/L) + b'_3 \sin(3\pi z/L) + \ldots + b'_n \sin(n\pi z/L) + \ldots,$
$C'(z) = \text{constant, or}$
$\quad = C_{eq}$

Similarly the generic solution to Fourier coefficients $\mathbf{b'_n}$:

$b'_n = (2 / L) \int C'(z) \sin(n\pi z/L)\, dz$
$\quad = (2\, C_{eq} / L) \int \sin(n\pi z/L)\, dz$
$\quad = (2\, C_{eq} / L)\, (L / n\pi)\, [-\cos(n\pi z/L)]$ where **z** ranges from 0 to L.
$\quad = (2\, C_{eq} / n\pi)\, [-\cos(n\pi) + \cos(0)]$

$b'_n = (2\, C_{eq} / n\pi)\, [(-1)^n + 1]$

In summary there are only two probable results:

$\mathbf{b'_n = (4\, C_{eq} / n\pi)}$ when **n** is odd and
$\mathbf{b'_n = 0}$ when **n** is even

For $n = 1$, $b'_1 = 4\, C_{eq} / \pi$
For $n = 2$, $b'_2 = 0$
For $n = 3$, $B'_3 = 4\, C_{eq} / 3\pi$
…

Substituting $\mathbf{b'_n}$ back into function **C'(z)** then:

$C'(z) = (4\, C_{eq} / \pi) \sin(\pi z/L) + (4\, C_{eq} / 3\pi) \sin(3\pi z/L) + (4\, C_{eq} / 5\pi) \sin(5\pi z/L) + \ldots +$
$\quad [4\, C_{eq} / (2n-1)\pi] \sin((2n-1)\pi z/L) + \ldots$

$\quad = (4\, C_{eq} / \pi) \sin(\pi z/L) +$
$\quad\quad (4\, C_{eq} / \pi) \{(1/3) \sin(3\pi z/L) +$
$\quad\quad\quad (1/5) \sin(5\pi z/L) + \ldots +$
$\quad\quad\quad [1 /(2n-1)] \sin((2n-1)\pi z/L) + \ldots\}$

$\quad = C'_1(z) + C'_{\ldots}(z)$

Where $C'_1(z) = (4\, C_{eq} / \pi) \sin(\pi z/L)$

Recalling:

$C(z) = b_1 \sin(\pi z/L) + b_2 \sin(2\pi z/L) + b_3 \sin(3\pi z/L) + \ldots + b_n \sin(n\pi z/L) + \ldots$

$\quad = (2 / L) \sin(\pi z/L) \int C(z) \sin(\pi z/L)\, dz + (2 / L) \sin(2\pi z/L) \int C(z) \sin(2\pi z/L)\, dz +$
$\quad\quad (2 / L) \sin(3\pi z/L) \int C(z) \sin(3\pi z/L)\, dz + \ldots + (2 / L) \sin(n\pi z/L) \int C(z) \sin(n\pi z/L)\, dz + \ldots$

$\quad = (2 / L) \sin(\pi z/L) \int C(z) \sin(\pi z/L)\, dz +$
$\quad\quad (2 / L) \{\sin(2\pi z/L) \int C(z) \sin(2\pi z/L)\, dz +$
$\quad\quad\quad \sin(3\pi z/L) \int C(z) \sin(3\pi z/L)\, dz + \ldots +$

$\sin(n\pi z/L) \int C(z) \sin(n\pi z/L)\, dz + \ldots \}$

$= C_1(z) + C_{\ldots}(z)$

Where $C_1(z) = (2 / L) \sin(\pi z/L) \int C(z) \sin(\pi z/L)\, dz$

An independent equation containing both the unknown properties and the known properties can now be formulated by equating the corresponding harmonic component of the series from both **C'(z)** and **C(z)** by taking the first term $\mathbf{C'_1(z) = C_1(z)}$:

$(4 C_{eq} / \pi) \sin(\pi z/L) = (2 / L) \sin(\pi z/L) \int C(z) \sin(\pi z/L)\, dz$

Finally

$C_{eq} = (\pi / 2L) \int C(z) \sin(\pi z/L)\, dz$

With the given values $\mathbf{c_1, c_2, \ldots c_i}$ corresponding to coordinates $\mathbf{z_1, z_2, \ldots z_i}$ the **unknown** equivalent warping constant $\mathbf{C_{eq}}$ can easily be solved for by numerical integration for ranges from **0** to **L** using either Trapezoidal Rule or Simpson's Rule

*To apply Simpson's rule properly, the number of equally spaced segments subdividing the girder must be an **odd figure**, defined by which the number of stations would then be an **even figure***

By means of successive trials coupled with increasing number of integration intervals, one reaches convergence to an equivalent value for the property of interest – in this case it's the warping constant.

Applying the same procedure to the full array of properties, the variable sectioned girder could be represented qualitatively by an equivalent member with just a single suite of properties.

*The word "equivalent" implies that the value arrived at is an **approximation** hence what demonstrated here is certainly not a replacement to the exact science.* But in a nutshell:

- Numerically, the accuracy of the application through **Fourier** Sine series expansion is highly dependent on the outcome from the integration $\int \mathbf{C(z) \sin(\pi z/L)\, dz}$. Much as we try, just beware that it is not a be-all cure-all way of handling the issue; Readers are encouraged to explore further in this topic

- Structurally, the level of authenticity from using equivalent property as pseudo-replacement to the variable properties is highly dependent on the innate geometry. Or in a way dependent on how irregular the contour or outline of the section geometry from end to end with respect to the *centerline of the girder*

Anyhow validity from using this approach in application for **CRG** or any other structure must be **fully justified** independently. Readers beware!!!

7.15 Variable Sectioned Member – An Application Example

By its not-so-friendly appearance, the expression $\mathbf{C_{eq} = (\pi / 2L) \int C(z) \sin(\pi z/L)\, dz}$ as concluded in the last section doesn't seem like anything practical, or else when judging by the appearance it must be "extremely" difficult to "solve" for an equivalent section property in an actual application.

But in reality, it is much more user-friendly than any other ways of doing or thinking in terms of everything extreme (say, the finite element way) comparatively.

What in need to show how convincing we meant is an example given as follows:

Example 7.3

Given Conditions:

(a) A stepped girder with L = 100 ^{ft} long (*with reference to Figure 7.2*)
(b) Flexural and torsionally simple supported at both ends
(c) Numerical integration at equally spaced 10 ^{ft} increment
(d) Define 11 stations with coordinates at **z** = 0', 10', 30' … **z** = 100'
(e) Define zone 1 through zone 5: Span 10', 10', 60', 10' and 10', respectively
(f) Corresponding zone warping constants: 40 in^6, 80 in^6, 120 in^6, 80 in^6 and 40 in^6

Required: Equivalent warping constant $\mathbf{C_{eq}}$

Solution:

Interestingly;

- Notice that the girder length given in the condition (a) is merely a token quantity through most part of the calculation until the end once all the dimensions were normalized for numerical integration

- Also the structural boundary conditions given in the condition (b) had no impact to the outcome; they were provided to prove a point

- The most important theme among all is the choice of solution scheme and the accuracy of the result, which is quite subjective

- ***Important to know: If the choice of integration is Simpson's Rule then the number of equally space segments must be even***

The numerical integration of a generic function $\int \mathbf{f(z)\ dz}$ by Simpson's Rule over even number of **n** equal spaces (or of **n + 1** stations) can be expanded into a series of algebraic terms (in the example **n** = 10) as follows:

$$\int \mathbf{f(z)\ dz} = (\Delta z / 3)\ [\mathbf{f(z_0) + 4\ f(z_1) + 2\ f(z_2) + 4\ f(z_3) + 2\ f(z_3) + \ldots + 4\ f(z_{n-1}) + f(z_n)}]$$

Where $\mathbf{\Delta z = L / n}$ and notice that the applicable term coefficients (or multipliers) were patterned in series {1, 4, 2, 4 … 2, 4, 2, 4, 1}

Prior to applying the technique, it is much better to tabulate all the information with proper identification.

Herein the goal is to apply the formula $\mathbf{C_{eq} = (\pi / 2L) \int C(z)\ sin(\pi z/L)\ dz}$ through Simpson's Rule, the critical portion of the formula is $\int \mathbf{C(z)\ sin(\pi z/L)\ dz}$, from which expanded into series:

$$\mathbf{C(z_0)\ sin(\pi z_0/L) + 4C(z_1)\ sin(\pi z_1/L) + 2C(z_2)\ sin(\pi z_2/L) + 4C(z_3)\ sin(\pi z_3/L) +}$$
$$\mathbf{\ldots + 2C(z_9)\ sin(\pi z_9/L) + 4C(z_{10})\ sin(\pi z_{10}/L) + C(z_{11})\ sin(\pi z_{11}/L)}$$

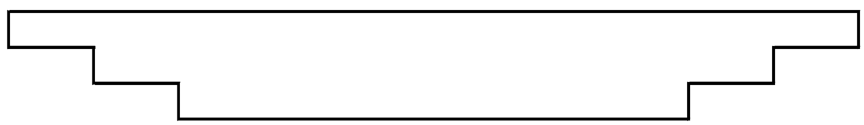
In that $\mathbf{C(z_0), C(z_1), C(z_2), C(z_3), \ldots}$ are the "known" warping constants corresponding to station coordinates $\mathbf{z_0, z_1, z_2, z_3,}$ and so forth

Look closely from Figure 7.2:

An important adjustment had been applied to the "given" warping constants (row 2) − that is using the "average" values (row 5) between "adjacent zones" as simplification of our numerical awkwardness if so opined

Picturing from a numerical point of view, what given are **collectively zone-based** as in step functions and what actually applied are **independently station-based** as in continuous functions

What becomes debatable is how to make the changeover of data from discrete gaps/steps into continuums. *Using straight average from the direct sum might not be the best way but is only one of many ways of making the transition*



1)	Zone ID	1	2	3	4	5
2)	Zone Warping Constant	40	80	120	80	40

3)	Station ID	1	2	3	4	5	6	7	8	9	10	11
4)	Z	0	L/10	2L/10	3L/10	4L/10	5L/10	6L/10	7L/10	8L/10	9L/10	L
5)	Averaged Nodal Warping Constant	40	60	100	120	120	120	120	120	100	60	40
6)	Sin(∏z/L)	0	0.309	0.588	0.809	0.951	1	0.951	0.809	0.588	0.309	0
7)	Simpson's Multiplier (A)	1	4	2	4	2	4	2	4	2	4	1
8)	(A)*Sin(∏z/L)	0	1.236	1.176	3.236	1.902	4	1.902	3.236	1.176	1.236	0

Figure 7.2 Equivalent Section Property − Example

It should be fairly straightforward following along with the definition/derivation/calculation row by row as shown in Figure 7.2. Finally:

$$\begin{aligned} \mathbf{C_{eq}} &= (\pi / 2L)\ (\mathbf{\Delta z / 3})\ [(40)(0) + (60)(1.236) + (100)(1.176) + (120)(3.236) + (120)(1.902) + \\ &\qquad (120)(4) + (120)(1.902) + (120)(3.236) + (100)(1.176) + (60)(1.236) + (40)\ (0)] \\ &= (\pi * 10 / 6 / 100)\ [0 + 74.16 + 11.76 + 388.32 + 288.24 + \\ &\qquad 480 + 288.24 + 388.32 + 11.76 + 74.16 + 0] \\ &= (0.05236)\ (2004.96) \\ &= 104.98 \end{aligned}$$

The result $\mathbf{C_{eq}}$ = 104.98 by all means is only an approximation, and one may never know what an exact value is; perhaps it may be improve by (1) using more sophisticated zone-to-station data transitioning strategy and (2) increasing the number of stations, and so forth.

7.16 Chapter Conclusion

CRG being dealt with in most "Rehab or Replacement/Upgrade Projects" demands extra attention such that we almost always have to operate in a more nit-picky mode than in caring for other breeds of structure.

As pointed out in previous Chapters that no two Engineers are alike in the depth of sense or wisdom over an identical structural subject or issue. Even if trained through issues and crises encountered in diverse practice routines, but there might still be areas of interest we were not very familiar with.

In this article several interesting topics were treated with modest amount of details, which might or might not be enough to demonstrate or be convincing enough that these "interesting things" were out there in the real world. Those subject matters being covered may be somewhat oddball, but knowing the fundamentals and how to work out the numbers for each and every brand of stress should be helpful in discerning whether it's acceptable to ignore certain categories of stress in our design.